Workshop on Algebraic Geometry and Homotopy Theory
October 3 - 4, 2019 - Radboud University Nijmegen

Program

October 3

09:30 - 10:30 Adrien Dubouloz
10:30 - 11:00 Coffee break
11:00 - 12:00 Victoria Hoskins
12:00 - 13:30 Lunch
13:30 - 14:30 Lenny Taelman
14:30 - 15:00 Coffee break
15:00 - 16:00 Emanuele Dotto
16:15 - 17:15 Maria Yakerson
17:15 Reception

October 4

09:30 - 10:30 Doosung Park
10:30 - 11:00 Coffee break
11:00 - 12:00 Lennart Meier
12:00 - 13:30 Lunch
13:30 - 14:30 Lie Fu
14:30 - 15:00 Coffee break
15:00 - 16:00 Marc Levine

Description

One aim of this workshop is to bring together researchers working on different questions lying on the overlap of algebraic geometry and homotopy theory. The workshop is partly funded by Radboud University and by the Radboud Excellence Initiative in connection with the Radboud Excellence visiting professorship held by Paul Arne Østvær.

Location

All talks take place in the *Global Lounge* of Radboud University Nijmegen, Thomas van Aquinostraat 1-A, Nijmegen, The Netherlands.

Organizers

This workshop is organized by Ben Moonen, Steffen Sagave, and Paul Arne Østvær.

Speakers, titles and abstracts

- Emanuele Dotto (University of Bonn / University of Warwick): *Grothendieck-Witt theory of quadratic functors*
  Abstract: I will talk about joint work with Calmès-Harpaz-Hebestreit-Land-Moi-Nardin-Nikolaus-Steimle on the Grothendieck-Witt spectrum of a stable infinity-category equipped with a non-degenerate quadratic functor. We will discuss examples, in particular how various quadratic functors on the category of perfect chain complexes give rise to various flavours of Grothendieck-Witt theory of rings. We will also establish a fiber sequence relating Grothendieck-Witt theory, K-theory and L-theory, and discuss Karoubi's periodicity theorem in this context.
- Adrien Dubouloz (Université de Bourgogne): *Affine bundles over and under affine quadrics*
  Abstract: I will discuss two constructions related to affine bundles and smooth affine quadrics. The first one provides an infinite countable family of pairwise non-isomorphic smooth rational real algebraic 4-folds whose complexifications are all isomorphic to the product of a smooth complex affine quadric surface with the affine plane. The second provides a realization of a moduli of $\mathbb{A}^1$-contractible complex exotic affine 3-spaces as algebraic quotients of suitable additive group actions on a same 4-dimensional smooth affine quadric.
- Lie Fu (Université Lyon/ RU Nijmegen): *Derived categories and motives of K3 surfaces*
  Abstract: Two complex K3 surfaces are called isogenous if their second rational cohomology are Hodge isometric. By the work of Huybrechts, an isogeny between two projective K3 surfaces implies that their Chow motives are isomorphic. We show that the converse does not hold and provide a Torelli-type theorem saying that being isogenous is equivalent to the condition that their Chow motives are isomorphic as Frobenius algebra objects. The (twisted) derived categories will play an essential role in the proof. If time permits, I will discuss the possibility to extend our result to higher dimensions. This is a joint work with Charles Vial, available on arXiv:1907.10868.
- Victoria Hoskins (FU Berlin): *On the motive of the moduli space of Higgs bundles*
  Abstract: In joint work with Simon Pepin Lehalleur, we study the motive of moduli space of semistable Higgs bundles of coprime rank and degree on a smooth projective curve in Voevodsky's triangulated category of motives. After a brief introduction to motives and some

preliminaries on Higgs bundles, we present two results. First we give a motivic non-abelian Hodge correspondence: the integral motives of the Higgs and de Rham moduli spaces are isomorphic. Second, when working with rational coefficients, we show that the motive of the Higgs moduli space is contained in the thick tensor subcategory generated by the motive of the curve; in fact, the argument involves a wall-crossing argument and a Harder-Narasimhan recursion on certain associated moduli stacks together with our work on a formula for the motive of the stack of vector bundles.

- Marc Levine (University of Duisburg-Essen): *GW-GW theory*
  Abstract: We give an overview of recent developments toward a refinement of Gromov-Witten theory that replaces the usual integer-valued invariants with quadratic forms, that is, elements in the Grothendieck-Witt ring of the field k over which the particular problem is defined. One should recover the usual integer invariants via the rank map, while over the reals, one should recover Gromov-Witten invariants in real algebraic geometry via the signature. Over other fields, one should get congruences (modulo a power of 2) for solutions to the geometric problem. Up to now, one uses methods from algebraic geometry, motivic homotopy theory and the theory of quadratic forms. This is in part a joint work with Jesse Kass, Jake Solomon and Kirsten Wickelgren.
- Lennart Meier (University of Utrecht): *When do derived stacks behave like affine schemes?*
  Abstract: Affine schemes can be characterized as those schemes whose category of quasi-coherent sheaves is equivalent to modules over the global sections over its structure sheaf. The analogous fact is not true if we move to derived algebraic geometry and allows sheaves of differential graded algebras or E-infinity ring spectra as structure sheaves. For example, quasi-affine schemes behave in the sense above like affine schemes in derived algebraic geometry. But this is not the end of the story. If a derived stack has an even-periodic structure sheaf, this determines a map from its underlying stack into the moduli stack of formal groups. In joint work with Mathew we have shown that it suffices that this map is quasi-affine. A major example is a certain even-periodic sheaf of E-infinity rings on the moduli stack of elliptic curves, whose global sections are the spectrum of topological modular forms.
- Doosung Park (University of Zurich): *Logarithmic motives over a field*
  Abstract: There are several cohomology theories over a field like Hodge cohomology theory that are not A1-invariant but still having other fundamental properties like the Projective bundle formula. These are not representable in DM. I will explain how to extend DM to include them using logarithmic geometry and cube-invariance. Some fundamental properties like Gysin triangles and blow-up triangles will be also discussed. This is joint with Federico Binda and Paul Arne Østvær.
- Lenny Taelman (University of Amsterdam): *Derived equivalences of hyperkähler varieties*
  Abstract: In this talk we consider auto-equivalences of the bounded derived category $D(X)$ of coherent sheaves on a smooth projective complex variety X. By a result of Orlov, any such auto-equivalence induces an (ungraded) automorphism of the singular cohomology $H(X,\mathbb{Q})$. If X is a K3 surface, then work of Mukai, Orlov, Huybrechts, Macrì and Stellari completely describes the image of the map $\rho_X : \mathrm{Aut}\, D(X) \to \mathrm{Aut}\,(H(X,\mathbb{Q}))$. We will study the image of ρ_X for higher-dimensional hyperkähler varieties. An important tool is a certain Lie algebra acting on $H(X,\mathbb{Q})$, introduced by Verbitsky, Looijenga and Lunts. We show that this Lie algebra is a derived invariant, and use this to study the image of ρ_X.
- Maria Yakerson (University of Regensburg): *Modules over algebraic cobordism*
  Abstract: When k is a field with resolution of singularities, it is known that Voevodsky's category of motives DM(k) is equivalent to the category of modules over the motivic cohomology spectrum HZ. This means that a structure of an HZ-module on a motivic spectrum is equivalent to a structure of transfers in the sense of Voevodsky. In this talk, we will discuss an analogous result for modules over the algebraic cobordism spectrum MGL. Concretely, a structure of an MGL-module is equivalent to a structure of coherent transfers along finite syntomic maps, over arbitrary base scheme. Time permitting, we will see a generalization of this result to modules over other motivic Thom spectra, such as the algebraic special linear cobordism spectrum MSL. This is joint work with Elden Elmanto, Marc Hoyois, Adeel Khan and Vladimir Sosnilo.