EAST Preparatory talks

Talk 1. Abelian categories, derived functors, derived categories

Define the notions of abelian category, additive exact functor (left-exact and right-exact), injective resolutions, categories with enough injectives (example: modules over a ring). Define the functors R^iF for $F : \mathcal{A} \to \mathcal{B}$ a left-exact functor, where $\mathcal{A}$ has enough injectives. Long exact sequence induced by short exact sequences using R^iF. Mention dual constructions: projective resolutions, L^iF.

Introduce the category of chain complexes $C(\mathcal{A})$, the homotopy category $K(\mathcal{A})$ defined by taking as maps homotopy classes of morphisms, and the derived category $D(\mathcal{A})$ defined by formally inverting quasi-isomorphisms. Equivalence of categories from $D(\mathcal{A}) \simeq K(Inj(\mathcal{A}))$. Introduce the (total) right derived functor of a left exact functor between abelian categories. If time permitted, give some examples.

Some classical references are [Wei94], [GM99], but there are also many good and compact online notes on the topic.

Talk 2. Sheaf theory and sheaf cohomology

Recall presheaves, sheaves and sheafification for sheaves on a topological space with values on a category (for example sets, abelian groups, rings). Introduce Grothendieck sites, generalizing topological spaces for sheaf theory. Define sheaves on a site and relate to classical sheaves on topological spaces. Define direct and inverse image functors for a map $f : X \to Y$. Define global sections functor. Introduce flasque sheaves and acyclic resolutions. Comment on the existence of the Godement resolution. Define the derived functors Rf^* and $R\Gamma$ using Godement resolutions. Introduce sheaf cohomology.

Three classical books are: [KS06], [Ten75], [Ive86]. The recent notes [Sch25] are a good reference for sites.

Talk 3. Stable ∞-categories, spectra

Define pointed ∞-categories and then introduce stable ∞-categories as pointed infinity categories where where pushouts and pullbacks coincide. Sketch why the homotopy category of a stable ∞-category inherits the structure of a triangulated category, with distinguished triangles coming from cofiber sequences. Give examples: the derived category of an abelian category, spectra, modules over a ring spectrum.

The second part should be about spectra. Explain their universal property as stabilization of pointed spaces. Sketch the construction of tensor product of spectra as unique functor which preserves colimits in each variable and such that the suspension functor from spaces is symmetric monoidal. Time permitting, give some examples of ring spectra (Eilenberg-MacLane, topological K-theory, etc.).

References: [Lur], [Gre19]

Talk 4. Spectral sequences

Define spectral sequence, and explain the notions of convergence and abutment. Give examples, such as the Grothendieck spectral sequence, and the Adams spectral sequence. You do not have and shouldn't prove anything: the goal is only to familiarize the audience with the language of spectral sequences.

In the second part, define t-structures and recall the definition of t-structure homotopy groups. Give examples such as the derived category or the standard t-structure on spectra. Then, following "Higher Algebra" explain how a filtered object in stable infinity-category with a t-structure gives rise to a spectral sequence and identify its E_1-page. Give an example of a spectral sequence constructed in this way.

References: [Lur] for the general theory, [Fun17] for the Grothendieck spectral sequence, [Ada74] for the Adams spectral sequence. The recent paper [Ant24] might be helpful in building the right intuition, so is worth a look for the speaker, but the results themselves are probably too technical for an introductory talk.

Talk 5. E_∞-algebra structure on singular cochains

The objective of this talk is to introduce E_∞-algebras motivated by the fact that the singular cochains of any topological space carries such a structure, inducing the cup product in cohomology. Since the notion of E_∞-algebra needs the theory of operads, this talk should recall the basics on operads in order to introduce E_∞-operads. A good short reference of E_∞-algebras is Section 4 of [Man02]. Other references include [BF04], [Ber16], where the E_∞-structure on cochains is explained. If time permits, one could explain how this construction works and/or show how to compute Steenrod operations on cohomology using the E_∞-action.

References

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[Wei94] Charles A. Weibel, *An introduction to homological algebra*, Cambridge Studies in Advanced Mathematics, vol. 38, Cambridge University Press, Cambridge, 1994.