EAST 2026 – Preparatory Talks

Preliminary knowledge: We assume familiarity with basic algebraic topology, including CW complexes, fundamental groups, homology and cohomology, fiber bundles, and elementary category theory. The talks on ∞-categories and ∞-operads should emphasize examples, universal properties, and analogy with ordinary categories rather than model-dependent technical details.

Talk (1) **The language of ∞-categories (60 minutes)**

Give an informal introduction to ∞-categories as categories in which morphisms may be composed only up to coherent homotopy. Briefly mention quasi-categories and the homotopy-coherent nerve of a simplicial category, but do not spend much time on their technical definitions.

Introduce the mapping space $\mathrm{Map}_{\mathcal{C}}(X, Y)$ and the homotopy category $h\mathcal{C}$. Explain how equivalences and subcategories of an ∞-category are defined. Discuss the principal examples: the ∞-category of spaces $\mathcal{S}$, the ∞-category Cat_∞ of small ∞-categories, and localizations of ordinary or enriched categories at weak equivalences.

Introduce functor ∞-categories $\mathrm{Fun}(\mathcal{C}, \mathcal{D})$. Define limits and colimits by their universal properties. Similarly, explain adjunctions between ∞-categories, emphasizing their similarity to ordinary categorical adjunctions. Discuss the presheaf category $\mathrm{PSh}(\mathcal{C}) = \mathrm{Fun}(\mathcal{C}^{\mathrm{op}}, \mathcal{S})$ and the Yoneda embedding $\mathcal{C} \to \mathrm{PSh}(\mathcal{C})$. Explain restriction along a functor and its left and right adjoints, given by left and right Kan extension. State the pointwise formulas for Kan extensions, at least informally, and mention the naturality of the Yoneda embedding with respect to left Kan extension. Introduce over- and undercategories and explain that their mapping spaces can be described as fibers of maps between mapping spaces. Give a conceptual account of straightening and unstraightening: functors $\mathcal{C}^{\mathrm{op}} \to \mathrm{Cat}_\infty$ correspond to Cartesian fibrations over $\mathcal{C}$. As the main example, explain the equivalence $\mathrm{PSh}(X) \simeq \mathcal{S}/X$ for a space X.

Time permitting, mention that limits of ∞-categories have mapping spaces computed as limits of mapping spaces.

References: [Lur09, Lan21, Hau23, Hau25]

Talk (2) **Whitehead torsion and the s-cobordism theorem (60 minutes)**

Begin with the distinction between homotopy equivalence and simple homotopy equivalence. Recall elementary expansions and collapses of finite CW complexes and explain that simple homotopy theory records whether a homotopy equivalence can be constructed from such elementary moves.

For a connected finite CW complex X, describe the cellular chain complex of its universal cover as a finite chain complex of free $\mathbb{Z}[\pi_1 X]$-modules. Recall the group $K_1(R)$ of a ring and define the Whitehead group $\mathrm{Wh}(\pi) = K_1(\mathbb{Z}[\pi])/\langle \pm g \,|\, g \in \pi \rangle$. Define the torsion of a finite acyclic chain complex. Explain how a homotopy equivalence $f\colon X \to Y$ of finite pointed CW complexes gives rise to its Whitehead torsion $\tau(f) \in \mathrm{Wh}(\pi_1 Y)$. State the basic properties of Whitehead torsion: invariance, the composition formula, and the criterion that a homotopy equivalence is simple precisely when its torsion vanishes. Define the notion of h-cobordism and the associated

Whitehead torsion. State the s-cobordism theorem: for a compact smooth h-cobordism W^{n+1} between n-manifolds with $n \geq 5$, the cobordism is trivial (i.e., diffeomorphic to $M^n \times I$) if and only if its Whitehead torsion vanishes. Also state the classification form of the theorem, identifying h-cobordisms over a fixed manifold with elements of its Whitehead group.

References: [Lüc02, DK01]

Talk (3) **Basics of ∞-operads (60 minutes)**

Begin by recalling ordinary colored operads. Explain that an operad consists of a collection of colors, spaces of operations with several inputs and one output, symmetric group actions, units, and composition maps. Introduce ∞-operads as their homotopy-coherent analog, again emphasizing the conceptual structure rather than a specific technical model.

Discuss the basic examples: the associative operad $\mathbb{E}_1$, the commutative operad $\mathbb{E}_\infty$, and the little disks operads $\mathbb{E}_d$ interpolating between them.

Recall that a symmetric monoidal ∞-category may be regarded as a commutative monoid object in Cat_∞. For an ∞-category $\mathcal{C}$ with finite coproducts, discuss its cocartesian symmetric monoidal structure $\mathcal{C}^{\amalg}$. Describe the Day convolution symmetric monoidal structure on $\mathrm{PSh}(\mathcal{C})$ when $\mathcal{C}$ is symmetric monoidal.

For an ∞-operad $\mathcal{O}$ and a symmetric monoidal ∞-category $\mathcal{C}$, introduce the ∞-category $\mathrm{Alg}_{\mathcal{O}}(\mathcal{C})$ of $\mathcal{O}$-algebras in $\mathcal{C}$.

References: [Lur09, Lur17, Hau23]

Talk (4) **Spectra and stable homotopy theory (60 minutes)**

Introduce pointed ∞-categories and stable ∞-categories. The category of spectra should be the main example, but it is also helpful to briefly discuss the general stabilization process.

Define the homotopy groups of spectra and explain how spectra correspond to cohomology theories. Connect to the previous talk by introducing the smash product of spectra and $\mathbb{E}_d$-ring spectra, and by summarizing the recognition principle. Discuss examples, including Eilenberg–Mac Lane spectra and topological K-theory spectra.

If time permits, you could also briefly mention more important examples of spectra or frequently used tools such as localization.

References: [Lur17, Cno26]

Talk (5) **Construction of Waldhausen's K-theory via the $S_\bullet$-construction (60 minutes)**

Begin by introducing the notion of Waldhausen categories (called "categories with cofibrations and weak equivalences" in [Wal85]) and exact functors between them. Your examples should include the category of finitely generated projective modules over a ring R and the category $R_f(X)$ of finite retractive spaces relative to a fixed space X.

Introduce Waldhausen's $S_\bullet$-construction and use it to define the algebraic K-theory space of a Waldhausen category. Returning to your earlier examples, you should use it to define the algebraic K-theory (spaces and groups) of a ring and of a space. Briefly explain that one obtains the correct answer on K_0.

Discuss the additivity theorem in at least one of its equivalent incarnations (without proof), and outline how it can be used to build a delooping of the algebraic K-theory space of a Waldhausen category from the iterated $S_\bullet$-construction.

If time permits, you can end with a short overview of some of the fundamental theorems about algebraic K-theory, such as the Approximation Theorem and the Fibration Theorem.

References: [Wal85, Rog10, Wei13]

References

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