COMMENTS AND CORRECTIONS TO DIAGRAM SPACES AND SYMMETRIC SPECTRA (ADVANCES IN MATHEMATICS, 231 (2012), 2116–2193)

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This note contains corrections to a few statements in our article "Diagram spaces and symmetric spectra". None of these issues affect the main results of the paper.

(1) The notion of a *flat cofibration* in Definition 3.9 is, in the topological setting, inconsistent with the use of this term in the rest of the paper. The correct definition reads as follows:

Definition 3.9. A map of $\mathcal{I}$-spaces $X \to Y$ is a *flat cofibration* if the induced map $X(\mathbf{n}) \cup_{L_{\mathbf{n}}(X)} L_{\mathbf{n}}(Y) \to Y(\mathbf{n})$ is a cofibration in the fine model structure on $\mathcal{S}^{\Sigma_n}$ for all $\mathbf{n}$. It is a *positive flat cofibration* if it is a flat cofibration and in addition $X(\mathbf{0}) \to Y(\mathbf{0})$ is an isomorphism.

Indeed, the above definition is precisely the criterion resulting from Proposition 6.8 in the paper by setting $\mathcal{K} = \mathcal{I}$ and $\mathcal{A} = \Sigma$ (see Section 6.1 for a discussion of the fine model structure on $\mathcal{S}^{\Sigma_n}$). The two definitions agree in the simplicial setting as follows from the discussion in Remark 6.5.

The reason for this error in Definition 3.9 is that at some point during the writing, the use of the term *flat cofibration* got mixed up with the (different) notion of flatness used in the papers [1, 2]. This also applies to the flatness criterion in Proposition 3.12 which ensures that an $\mathcal{I}$-space X is flat in the sense of [1, 2], but not in the sense of the present paper. It is worth pointing out that the weaker flatness criterion in [1, 2] is sufficient to ensure that the $\boxtimes$-product is homotopically well-behaved. This is discussed in [2, Proposition 2.5].

(2) The sign functor $\mathrm{sgn} : \Sigma \to \{\pm 1\}$ introduced above (4.2) is strict monoidal but not symmetric monoidal as claimed. This does not lead to problems since strict monoidality is all that is needed for the definition of the functor sgn in (4.2).

(3) The positive level version of Proposition 8.2 is wrong. It states that if X is $\mathcal{A}$-relative cofibrant, then $X \boxtimes -$ preserves $\mathcal{A}$-relative level equivalences.

We give a counterexample in the case of the positive $\mathcal{I}$-model structure. Let $X = F_1^{\mathcal{I}}(*)$, and let $Y \to Y'$ be a positive level equivalence that fails to be an absolute level equivalence. Using Lemma 5.6 and the fact that the category $\mathbf{1} \sqcup - \downarrow \mathbf{1}$ is equivalent to the terminal category, it follows that there is a natural isomorphism $(F_1^{\mathcal{I}}(*) \boxtimes Y)(\mathbf{1}) \cong Y(\mathbf{0})$. Thus evaluating the map

$$(F_1^{\mathcal{I}}(*) \boxtimes Y) \to (F_1^{\mathcal{I}}(*) \boxtimes Y')$$

in level 1 provides a map of spaces that is isomorphic to $Y(\mathbf{0}) \to Y'(\mathbf{0})$. By our assumptions, this map is not a weak equivalence. Hence $X \boxtimes Y \to X \boxtimes Y'$ is not a positive level equivalence.

It is clear that the above counterexample does not apply if we are in an *absolute* setup, i.e., if we assume in addition that the subcategory $\mathcal{A}$ of

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$\mathcal{K}$ contains all objects of $\mathcal{K}$. When $\mathcal{A}$ satisfies this additional hypothesis, then the isomorphism of Lemma 5.6 shows that $F_{\mathbf{k}}^{\mathcal{K}}(L) \boxtimes -$ does preserve $\mathcal{A}$-relative level equivalences for all spaces L. Using the level version of Proposition 7.1, the argument given in the second part of the proof of Proposition 8.2 shows how the latter statement implies the one for a general $\mathcal{A}$-relative cofibrant X, that is, the functor $X \boxtimes -$ preserves $\mathcal{A}$-relative level equivalences.

It turns out that this issue with Proposition 8.2 has no consequences for the rest of the paper: The pushout-product axiom (Proposition 8.4) and the monoid axiom (Proposition 8.6) also hold for the positive level model structures since they can be checked on the generating acylic cofibrations. (Here the above counterexample does not lead to a contradiction since the generating acyclic cofibrations of the positive level model structures are in fact absolute level equivalences.) For the same reason, Proposition 9.3 and Lemma 9.5 continue to hold also for the positive level model structures. Moreover, we note that the proof of Proposition 9.13 only makes use of the absolute level version of Proposition 8.2 and is therefore not affected by the above problem.

(4) Page 2132, line -7: One α_1 should be an α_2.

References

[1] A. J. Blumberg, R. L. Cohen, and C. Schlichtkrull. Topological Hochschild homology of Thom spectra and the free loop space. *Geom. Topol.*, 14:1165–1242 (electronic), 2010.

[2] C. Schlichtkrull. The homotopy infinite symmetric product represents stable homotopy. *Algebr. Geom. Topol.*, 7:1963–1977, 2007.

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