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Winter term 2011/12

Graduate Seminar Topology S4D2
"The Steenrod algebra"

Tuesday 14-16, room 0.006

The action of cohomology operations provides a useful additional structure on the cohomology ring of a space. For a prime number p, the algebra of all stable cohomology operations on cohomology with $\mathbb{Z}/p$-coefficients is known as the *Steenrod algebra* and admits an explicit characterization in terms of generators and relations.

In this seminar, we will construct the Steenrod algebra for $p = 2$, verify its properties, and give some applications. We will mostly follow the book *Cohomology operations and applications in homotopy theory* by Mosher and Tangora [MT68] which reappeared in print recently.

Schedule

Talk 1, 2011/10/11: *The Eilenberg-Zilber Theorem, acyclic models, and the cup product via diagonal approximation* ([ML95, VIII §7 - §9], [Bre97, VI §4])

Talk 2, 2011/10/18: *Construction of the $\cup_i$-products* [MT68, Chapter 2, p. 12-16] and [Bre97, VI §16])

Talk 3, 2011/10/25: *Construction of the Squares* [MT68, Chapter 2, p. 16-21]

Talk 4, 2011/11/08: *Properties of the Squares* ([MT68, Chapter 3, p. 22-28])

Talk 5, 2011/11/15: *The Adem relations* ([MT68, Chapter 3, p. 29-31] and [BM82])

Talk 6, 2011/11/22: *The Hopf Invariant* ([MT68, Chapter 4, p. 33-38])

Talk 7, 2011/11/29: *The Steenrod Algebra* ([MT68, Chapter 5, p. 45-50])

Talk 8, 2011/12/06: *The dual of the Steenrod Algebra* ([MT68, Chapter 5, p. 50-57] and [Mil58])

Talk 9, 2011/12/13: *Exact Couples* ([Hat, p. 1-7])

Talk 10, 2011/12/20: *The Serre spectral sequence* ([Hat, p. 8-13])

Talk 11, 2012/01/10: *Transgression and the cohomology spectral sequence of a fibration* ([MT68, Chapter 8, p. 80-81])

Talk 12, 2012/01/17: *Computation of the cohomology ring $H^*(K(\mathbb{Z}/2,2);\mathbb{Z}/2)$* ([MT68, Chapter 9, p. 83-88])

Talk 13, 2012/01/24: *Computation of the cohomology ring $H^*(K(\mathbb{Z}/2,q);\mathbb{Z}/2)$* ([MT68, Chapter 9, p. 88-92])

References

[BM82] S. R. Bullett and I. G. Macdonald. On the Adem relations. *Topology*, 21(3):329–332, 1982. DOI:10.1016/0040-9383(82)90015-5.

[Bre97] Glen E. Bredon. *Topology and geometry*, volume 139 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1997. Corrected third printing of the 1993 original.

[Hat] Allen Hatcher. Spectral sequences in algebraic topology. Book project, available at http://www.math.cornell.edu/~hatcher/SSAT/SSATpage.html.

[Mil58] John Milnor. The Steenrod algebra and its dual. *Ann. of Math. (2)*, 67:150–171, 1958. DOI:10.2307/1969932.

[ML95] Saunders Mac Lane. *Homology*. Classics in Mathematics. Springer-Verlag, Berlin, 1995. Reprint of the 1975 edition.

[MT68] Robert E. Mosher and Martin C. Tangora. *Cohomology operations and applications in homotopy theory*. Harper & Row Publishers, New York, 1968.