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Summer term 2012

Seminar S4D4
"Model Categories"

Tuesday 12-14, room 0.006

In this seminar we study *model categories*, an axiomatic framework for homotopy theory introduced by Quillen [Qui67]. By definition, a model category is a category together with three preferred classes of morphisms that satisfy a list of axioms. Topological spaces, simplicial sets, and chain complexes of modules over a ring are examples for categories which can be equipped with the structure of a model category.

Our main references will be the survey article [DS95] by Dwyer and Spalinski and Hovey's book [Hov99].

Schedule

Talk 1, 03.04.2012: *Axioms, homotopy relation, and the model structure on categories* ([DS95, §3 and §4], [Hov99, §1], and [Rez] for the example)

Talk 2, 10.04.2012: *Chain complexes of modules* ([DS95, §7] and [Hov99, §2.3])

Talk 3, 17.04.2012: *Quillen functors and the homotopy category* ([DS95, §5 and §9] and [Hov99, §1.2 and §1.3])

Talk 4, 24.04.2012: *Topological spaces with the Quillen model structure* ([DS95, §8] and [Hov99, §2.4])

Talk 5, 08.05.2012: *Topological spaces with the Strøm model structure* ([Str72, Col06a, Col06b])

Talk 6, 15.05.2012: *Simplicial sets I* ([Hov99, §3] and [GJ99])

Talk 7, 22.05.2012: *Simplicial sets II* ([Hov99, §3] and [GJ99])

Talk 8, 05.06.2012: ([GS07, §4.2 and Theorem 4.17] or [GJ99, II.§2-§4 and Example II.5.2])

Talk 9, 12.06.2012: *Model categories of algebras* ([SS00])

Talk 10, 19.06.2012: *Homotopy colimits* ([DS95, §10] [Hir03, §18])

Talk 11, 26.06.2012: *Sequential spectra I* ([BF78] and [Bou01, §9])

Talk 12, 03.07.2012: *Sequential spectra II* ([BF78] and [Bou01, §9])

Talk 13, 10.07.2012: *The model structure on simplicial categories* ([Ber07])

Talk 14, 10.07.2012: *The Joyal model structure on simplicial sets* ([DS11, Appendix C])

References

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[DS11] Daniel Dugger and David I. Spivak. Mapping spaces in quasi-categories. *Algebr. Geom. Topol.*, 11(1):263–325, 2011. DOI:10.2140/agt.2011.11.263.

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