Abstract. Consider a complex affine variety $\tilde{V}$ and a real analytic Zariski-dense submanifold $V$ of $\tilde{V}$. We compare modules over the ring $\mathcal{O}(\tilde{V})$ of regular functions on $\tilde{V}$ with modules over the ring $C^\infty(V)$ of smooth complex valued functions on $V$.

Under a mild condition on the tangent spaces, we prove that $C^\infty(V)$ is flat as a module over $\mathcal{O}(\tilde{V})$. From this we deduce a comparison theorem for the Hochschild homology of finite type algebras over $\mathcal{O}(\tilde{V})$ and the Hochschild homology of similar algebras over $C^\infty(V)$.

We also establish versions of these results for functions on $\tilde{V}$ (resp. $V$) that are invariant under the action of a finite group $G$. As an auxiliary result, we show that $C^\infty(V)$ has finite rank as module over $C^\infty(V)^G$.

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Introduction

Let $\tilde{V}$ be a complex affine variety and let $V \subset \tilde{V}$ be a smooth submanifold. The general goal of this paper is to compare modules over the algebra of regular functions $\mathcal{O}(\tilde{V})$ with modules over the algebra of (complex-valued) smooth functions $C^\infty(V)$. On the algebraic side $\tilde{V}$ may be singular. On the smooth side we allow minor singularities via a smooth action of a finite group $G$, so that we actually consider smooth functions on an orbifold $V/G$. The precise conditions needed for our results are:

**Conditions A.**

(i) $V$ is a real analytic Zariski-dense submanifold of $\tilde{V}$,

(ii) the action of $G$ on $V$ extends to an action of $G$ on $\tilde{V}$, by algebraic automorphisms,

(iii) for all $v \in V$, $T_v(\tilde{V}) = T_v(V) \otimes_\mathbb{R} \mathbb{C}$.

Typical examples come from real forms of $\tilde{V}$ (but maybe not all real forms qualify). Sometimes (iii) can be replaced by

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(iii’) $G$ acts freely on $V$ (e.g. $G = 1$) and for each $v \in V$, the real vector space $T_v(V)$ spans the complex vector space $T_v(\tilde{V})$.

The assumptions (i) and (ii) guarantee that $O(\tilde{V})$ embeds $G$-equivariantly in $C^\infty(V)$. Either of (iii) and (iii’) entails that at every point of $V$ the formal completion of $O(\tilde{V})$ is a subalgebra of the formal completion of $C^\infty(V)$. Under condition (iii’), $V/G$ can be endowed with the structure of a smooth manifold.

**Theorem B.** (see Theorem 1.5) Assume that (i), (ii) and (iii) or (iii’) hold. Then $C^\infty(V)^G$ is flat over $O(\tilde{V})^G$.

The proof runs mainly via formal completions of $C^\infty(V)^G$-modules. We remark that $C^\infty(V)^G$ can be substantially more complicated than $C^\infty(V)$, for instance its Hochschild homology can be nontrivial in degrees beyond the dimension of $V$.

Our main application of this result is to the Hochschild homology of finite type algebras, as studied in [KNS]. Recall that a unital algebra $A$ (not necessarily commutative) is a finite type $O(\tilde{V})^G$-algebra if an algebra homomorphism from $O(\tilde{V})^G$ to the centre of $A$ is given, and makes $A$ into a finitely generated $O(\tilde{V})^G$-module.

Under the above conditions $C^\infty(V)^G \otimes_{O(\tilde{V})^G} A$ is a Fréchet algebra (this is why we need $V$ to be real-analytic). Furthermore it is finitely generated as $C^\infty(V)^G$-module, so it is reasonable to regard it as a smooth finite type algebra.

**Theorem C.** (see Theorem 2.3) Assume that (i), (ii) and (iii) from Conditions A hold. There is a natural isomorphism of Fréchet $C^\infty(V)^G$-modules

$$C^\infty(V)^G \otimes_{O(\tilde{V})^G} HH_n(A) \rightarrow HH_n\left(C^\infty(V)^G \otimes_{O(\tilde{V})^G} A\right).$$

We note that on the right hand side the Hochschild homology involves the topology of the algebra, via the complete projective tensor product of Fréchet spaces. Theorem C is a smooth version of an earlier result with formal completions [KNS].

To improve the effect of Theorem C for the computation of the Hochschild homology of smooth finite type algebras, we make its left hand side explicit in some cases. That involves one result of general nature:

**Theorem D.** (see Theorem 3.1) $C^\infty(V)$ is finitely generated as $C^\infty(V)^G$-module.

Let $\Omega^n(\tilde{V})$ be the $O(\tilde{V})$-module of algebraic differential $n$-forms on $\tilde{V}$ and denote the $C^\infty(V)$-module of smooth $n$-forms on $V$ by $\Omega^n_{sm}(V)$.

**Theorem E.** (a special case of Lemma 3.4) Suppose that (i), (ii) and (iii) from Conditions A hold. There is a natural isomorphism of Fréchet $C^\infty(V)^G$-modules

$$C^\infty(V)^G \otimes_{O(\tilde{V})^G} \Omega^n(\tilde{V}) \cong \Omega^n_{sm}(V).$$

Combining Theorems C and E we find
\[ HH_n(C^\infty(V)) = HH_n\left(C^\infty(V)^G \otimes_{\mathcal{O}(\tilde{V})^G} \mathcal{O}(\tilde{V})\right) \cong C^\infty(V)^G \otimes_{\mathcal{O}(V)^G} HH_n(\mathcal{O}(\tilde{V})). \]

Here we may remove the singular locus of \( \tilde{V} \), because it does not meet \( V \). Then \( \tilde{V} \) is nonsingular, so we can invoke the Hochschild–Kostant–Rosenberg theorem \cite{Lod}, Theorem 3.4.4. Another application of Theorem E to the right hand side, yields natural isomorphisms
\[ HH_n(C^\infty(V)) \cong C^\infty(V)^G \otimes_{\mathcal{O}(V)^G} \Omega^n(\tilde{V}) \cong \Omega^n_{sm}(V). \]

In this way we recover Connes’ version of the Hochschild–Kostant–Rosenberg theorem \cite{Con}, for the Hochschild homology of the Fréchet algebra of smooth functions on a real analytic manifold \( V \). Obviously that would be an extremely roundabout proof – the advantage of our methods is rather that they apply in much larger generality. In particular our results will be useful for the computation of the Hochschild homology of the Harish-Chandra–Schwartz algebra of a reductive \( p \)-adic group, for which we refer to \cite{Sol2}.

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1. Flatness of smooth functions as module over regular functions

Let \( V \) be a smooth manifold (without boundary) and let \( G \) be a finite group acting on \( V \) by diffeomorphisms. Consider the algebra \( C^\infty(V)^G \) of \( G \)-invariant smooth complex-valued functions on \( V \). For each \( v \in V \) we have the closed maximal ideals of functions vanishing at \( v \): \( I_v \subset C^\infty(V) \) and \( I_{Gv} \subset C^\infty(V)^G \). Let \( FP_v \) be the Fréchet algebra of formal power series on an infinitesimal neighborhood of \( v \) in \( V \). Then
\[ FP_v \cong \varprojlim_n C^\infty(V)/I_v^n \quad \text{and} \quad FP_{Gv} \cong \varprojlim_n C^\infty(V)^G/I_{Gv}^n. \]

By a theorem of Borel (see \cite{Tou, Théorème IV.3.1 and Remarque IV.3.5} or \cite{MeVo, Theorem 26.29}) the Taylor series map \( C^\infty(M) \to FP_v \) is surjective. Its kernel is the module \( I_v^\infty \) of functions that are flat at \( v \). For \( G \)-invariant functions that becomes an isomorphism
\[ FP_{Gv}^G \cong C^\infty(V)^G/I_{Gv}^\infty. \]

For any Fréchet \( C^\infty(V)^G \)-module \( M \) we can form the “formal completion” at \( v \):
\[ \hat{M}_{Gv} := \varprojlim_n M/\langle I_{Gv} \rangle M = FP_{Gv} \otimes_{C^\infty(V)^G} M \cong M/\langle I_{Gv} \rangle M. \]

In contrast with the algebraic setting, \( \hat{M}_{Gv} \) is actually a quotient rather than a completion of \( M \).

**Lemma 1.1.** Let \( M \) be a finitely generated Fréchet \( C^\infty(V)^G \)-module. Let \( M_1 \) and \( M_2 \) be closed \( C^\infty(V)^G \)-submodules of \( M \), such that \( M_1 \supset M_2 \) and \( \hat{M}_{Gv} = \hat{M}_{2Gv} \) for all \( v \in V \). Then \( M_1 = M_2 \).

**Proof.** By assumption there exists a finitely generated free \( C^\infty(V)^G \)-module \( N \) and a surjective homomorphism of Fréchet \( C^\infty(V)^G \)-modules \( p : N \to M \). By the
continuity of \( p, N_i := p^{-1}(M_i) \) is a closed \( C^\infty(V)^G \)-submodule of \( N \). For any \( v \in V \) we have
\[
\frac{N_1}{N_2} = \frac{M_1}{M_2} = 0.
\]
From that and (1.2) we deduce
\[
(1.3) \quad \frac{N_1}{N_2} = \frac{I_{Gv}^\infty N_1}{N_2} = \frac{I_{Gv}^\infty N_1}{N_2} + \frac{N_2}{N_2}.
\]
By Whitney’s spectral theorem \cite[Corollaire V.1.6]{Tou}, \( I_{Gv}^\infty = (\bigcap_{v' \in Gv} I_{v'}^\infty)^G \) is closed in \( C^\infty(V)^G \). As \( N_1 \) is closed in \( N \), (1.3) equals
\[
(\frac{I_{Gv}^\infty N \cap N_1}{N_2} + \frac{N_2}{N_2}) = (\frac{I_{Gv}^\infty N + N_2}{N_2})/N_2.
\]
This holds for all \( v \in V \), so \( N_1 \) is contained in \( \bigcap_{v \in V} I_{Gv}^\infty N + N_2 \). Again by Whitney’s spectral theorem \cite[Corollaire V.1.6]{Tou}, the latter equals the closure of \( N_2 \) in the free module \( N \). Since \( N_2 \) is closed, we conclude that \( N_1 \subseteq N_2 \). Hence \( N_1 = N_2 \) and \( M_1 = M_2 \).

In this context it is useful to mention the following slight generalization of a result of Malgrange \cite[Corollaire VI.1.8]{Tou}.

**Theorem 1.2.** Assume that \( V \) is real analytic, and let \( M \) be a \( C^\infty(V)^G \)-submodule of \( (C^\infty(V)^G)^r \) generated by finitely many real-analytic \( G \)-invariant functions from \( V \) to \( \mathbb{C} \). Then \( M \) is closed in \( (C^\infty(V)^G)^r \).

**Proof.** Let \( \{f_i\} \) be a finite set of analytic \( G \)-invariant functions from \( V \) to \( \mathbb{C} \). By \cite[Corollaire VI.1.8]{Tou} they generate a closed \( C^\infty(V) \)-submodule \( M' \) of \( C^\infty(V)^r \).

Assume that the \( f_i \) generate \( M \) as \( C^\infty(V)^G \)-module. Write \( p_G = |G|^{-1} \sum_{g \in G} g \), an idempotent in \( \mathbb{C}[G] \). Clearly \( M \subseteq M' \cap (C^\infty(V)^G)^r \). On the other hand
\[
M = \sum f \cdot C^\infty(V)^G f = \sum f \cdot (p_G C^\infty(V) f) = p_G \sum f \cdot (p_G C^\infty(V) f)
\]
Hence \( M = M' \cap (C^\infty(V)^G)^r \), which is closed in \( (C^\infty(V)^G)^r \) because \( M' \) is closed in \( C^\infty(V)^r \).

Let \( \tilde{V} \) be a complex affine \( G \)-variety and recall the Conditions \( \mathbb{A} \).

**Lemma 1.3.** Assume (i) and (ii) from Conditions \( \mathbb{A} \) and let \( M \) be a finitely generated \( \mathcal{O}(\tilde{V}) \)-module. The \( C^\infty(V)^G \)-modules
\[
C^\infty(V)^G \otimes_{\mathcal{O}(\tilde{V})^G} M, \quad F_{\mathcal{O}(\tilde{V})^G}^G \otimes_{\mathcal{O}(\tilde{V})^G} M \quad \text{and} \quad I_{Gv}^\infty(C^\infty(V)^G \otimes_{\mathcal{O}(\tilde{V})^G} M)
\]
are Fréchet. The first two are generated by a finite subset of \( M \).

**Proof.** Any finite set of generators of \( M \) as \( \mathcal{O}(\tilde{V}) \)-module also generates the first two \( C^\infty(V)^G \)-modules under consideration. By \cite[(30) and subsequent lines]{OpSo}, every finitely generated \( F_{\mathcal{O}(\tilde{V})^G}^G \)-module is Fréchet, so in particular \( F_{\mathcal{O}(\tilde{V})^G}^G \otimes_{\mathcal{O}(\tilde{V})^G} M \).

Pick \( r \in \mathbb{Z}_{>0} \) and a \( \mathcal{O}(\tilde{V})^G \)-submodule \( N \) of \( (\mathcal{O}(\tilde{V})^G)^r \) such that \( M \cong (\mathcal{O}(\tilde{V})^G)^r/N \).

The kernel of the surjective homomorphism of \( C^\infty(V)^G \)-modules
\[
(1.4) \quad C^\infty(V)^G \otimes_{\mathcal{O}(\tilde{V})^G} (\mathcal{O}(\tilde{V})^G)^r \rightarrow C^\infty(V)^G \otimes_{\mathcal{O}(\tilde{V})^G} (\mathcal{O}(\tilde{V})^G)^r/N = C^\infty(V)^G \otimes_{\mathcal{O}(\tilde{V})^G} M
\]
Lemma 1.4. Assume that (i), (ii) and either (iii) or (iii') from Conditions A hold and let $M \subset M'$ be $\mathcal{O}(\hat{V})^G$-modules. Then the natural map

$$(C^\infty(\hat{V})^G \otimes \mathcal{O}(\hat{V})^G)_{\hat{V}} \to (C^\infty(\hat{V})^G \otimes \mathcal{O}(\hat{V})^G)_{\hat{V}}$$

is injective.

Proof. Recall that the formal completion of the $\mathcal{O}(\hat{V})^G$-module $M$ at $Gv \in V/G$ is defined as

$$(1.6) \quad \hat{M}_{Gv} = \varprojlim_n M/(P^n_G \cap \mathcal{O}(\hat{V})^G)M.$$ 

Let $\hat{FP}_v$ be the formal completion of $\mathcal{O}(\hat{V})$ at $v \in V$. One checks readily that

$$(1.7) \quad (C^\infty(\hat{V})^G \otimes \mathcal{O}(\hat{V})^G)_{\hat{V}} \cong \hat{FP}_v \otimes \hat{M}_{Gv}.$$ 

By the exactness of the formal completion functor $\hat{FP}_v$, $\hat{M}_{Gv}$ is a $\hat{FP}_v$-submodule of $\hat{M}'_{Gv}$.

Suppose that (iii) holds. Then $FP_v \cong \hat{FP}_v$ as $G_v$-representations, so $FP_v \cong \hat{FP}_v$. With $\hat{FP}_v$, that immediately implies the statement.

Suppose that (iii') holds, so that $G_v = 1$. Pick a complex vector space $T$ such that

$$(1.8) \quad T_v(V) \otimes \mathbb{C} = T_v(\hat{V}) \oplus T.$$ 

Let $\{z_1, \ldots, z_d\}$ be a basis of $T_v(V)^*$ and let $\{w_1, \ldots, w_{\dim V-d}\}$ be a basis of $T^*$. There are isomorphisms of Fréchet algebras

$$(1.9) \quad FP_v \cong \mathbb{C}[[z_1, \ldots, z_d, w_1, \ldots, w_{\dim V-d}]] \cong \mathbb{C}[[z_1, \ldots, z_d]] \otimes \mathbb{C}[[w_1, \ldots, w_{\dim V-d}]] \cong \hat{FP}_v \otimes \mathbb{C}[[T^*]].$$

The module $\hat{FP}_v \otimes \mathbb{C}[T^*]$ is free over $\hat{FP}_v$, so in particular flat. From $\hat{FP}_v$ we see that $FP_v$ is the formal completion of $FP_v \otimes \mathbb{C}[T^*]$ at $0 \in T$, and is flat over $\hat{FP}_v \otimes \mathbb{C}[T^*]$. Hence $FP_v = FP_v^{\hat{FP}_v}$ is flat over $\hat{FP}_v = \hat{FP}_v^{\hat{FP}_v}$, together with $\hat{FP}_v = \hat{FP}_v$ and $\hat{FP}_v$, that implies the statement of the lemma. \qed
We note that Lemma 1.4 may become false if assume only (i), (ii) and (iii') in a weaker version without the freedom of the $G$-action. For example, take $V = V = C$, on which $G = \{1, -1\}$ acts by multiplication. For $v = 0$ we have
\[ FP_{v}^{G_{v}} = C[[z^{2}]] \quad \text{and} \quad FP_{v}^{G_{v}} = C[[z, \bar{z}]]^{G} = C[[z^{2}, \bar{z}^{2}, z\bar{z}]]. \]
Here $FP_{v}^{G_{v}}$ is not flat over $FP_{v}$, and then we see from (1.7) that Lemma 1.4 fails.

Our main result about flatness generalizes the flatness of $C^{\infty}(\tilde{V})$ over $O(\tilde{V})$. As pointed out in an answer to a question on MathOverflow\footnote{mathoverflow.net/questions/226136/is-the-sheaf-of-smooth-functions-flat} that case can be shown quickly with results of Malgrange [Mal] about complex analytic functions.

**Theorem 1.5.** Assume that (i), (ii) and either (iii) or (iii') from Conditions A hold. Then $C^{\infty}(V)^{G}$ is flat as $O(\tilde{V})^{G}$-module. In particular the functor (1.5) is exact.

**Proof.** According to [Eis, Proposition 6.1], flatness can be checked by testing it with finitely generated modules. Let $M \subset M'$ be finitely generated $O(\tilde{V})^{G}$-modules. We need to show that the natural map
\[ \mu : C^{\infty}(V)^{G} \otimes_{O(\tilde{V})^{G}} M \to C^{\infty}(V)^{G} \otimes_{O(\tilde{V})^{G}} M' \]
is injective. We want to apply Lemma 1.1 inside the domain of $\mu$, which by Lemma 1.3 has the right properties. The submodules will be $M_{0} = 0$ and $M_{1} = \ker(\mu)$, which is a closed submodule of the domain because $\mu$ is continuous and $C^{\infty}(V)^{G}$-linear. Lemma 1.1 yields the desired conclusion $\ker(\mu) = 0$, provided we can check that all formal completions of the $C^{\infty}(V)^{G}$-module $\ker(\mu)$ are zero.

From Lemma 1.3 we know that $\mu_{G_{v}}$ is injective. We would like to apply the exactness of the formal completion functor from [OpSo, Theorem 2.5] to
\[ 0 \to \ker(\mu) \to C^{\infty}(V)^{G} \otimes_{O(\tilde{V})^{G}} M \to C^{\infty}(V)^{G} \otimes_{O(\tilde{V})^{G}} M', \]
but unfortunately $\ker(\mu)$ could be a topological vector space of a more general kind than allowed by [OpSo, Theorem 2.5]. It turns out that we can still use the proof of [OpSo, Theorem 2.5], which relies on technical constructions in [MeTo, Chapitre 1].

Consider an element of $\ker(\mu)^{G_{v}}$ represented by $m \in \ker(\mu) \subset C^{\infty}(V)^{G} \otimes_{O(\tilde{V})^{G}} M$. By the injectivity of $\mu_{G_{v}}$ and (1.2), $m$ belongs to
\[ I_{G_{v}}^{\infty}(C^{\infty}(V)^{G} \otimes_{O(\tilde{V})^{G}} M). \]
Here taking the closure is superfluous, for by Lemma 1.3 it is already a closed subspace of $C^{\infty}(V)^{G} \otimes_{O(\tilde{V})^{G}} M$. Hence there are finitely many $f_{j} \in I_{G_{v}}^{\infty}$ and $m_{j} \in M$ such that $m = \sum_{j} f_{j} \otimes m_{j}$. By [MeTo, p. 183] there exists a $\psi \in I_{v}^{\infty}$ such that $f_{j}/\psi \in I_{v}^{\infty} \subset C^{\infty}(V)$ for all $j$.

By averaging over $G$, we may assume that $\psi \in I_{G_{v}}^{\infty} \subset C^{\infty}(V)^{G}$. Then
\[ m/\psi = \sum_{j} f_{j}/\psi \otimes m_{j} \in C^{\infty}(V)^{G} \otimes_{O(\tilde{V})^{G}} M. \]
is well-defined. The construction in \cite[pp. 184]{MeTo} yields a sequence of functions \( \epsilon_i \in I_v^\infty \) such that \( \sum_i \epsilon_i = \psi \) and \( \epsilon_i / \psi \in I_v^\infty \). Again, by averaging over \( G \) we assume that the \( \epsilon_i \) are \( G \)-invariant, so that \( (\epsilon_i / \psi)m \in I_v^\infty \ker(\mu) \). Now we can write
\[
m = \psi \cdot m / \psi = \sum_i \epsilon_i \cdot m / \psi = \sum_i (\epsilon_i / \psi) \cdot m \in I_v^\infty \ker(\mu).
\]
The sums converge by the equalities (although \( \sum_i (\epsilon_i / \psi) \) does not converge). Hence \( m = 0 \) in \( \hat{\ker(\mu)} \), and \( \hat{\ker(\mu)} = 0 \).

\[\Box\]

2. Finite type algebras and their smooth versions

We will apply Theorem 1.5 to finite type algebras. By an \( \mathcal{O}(\tilde{V})^G \)-algebra we mean a (not necessarily unital) algebra \( A \) together with a unital algebra homomorphism from \( \mathcal{O}(\tilde{V})^G \) to the centre of the multiplier algebra of \( A \). Recall from \cite{KNS} that \( A \) has finite type (as \( \mathcal{O}(\tilde{V})^G \)-algebra) if it is finitely generated as module over \( \mathcal{O}(\tilde{V})_G \). The homology and representation theory of such algebras was studied in \cite{KNS}. We want to compare \( A \) and
\[
C^\infty(V)^G \otimes_{\mathcal{O}(\tilde{V})^G} A.
\]

By Lemma 1.3 this is a Fréchet algebra, and it is finitely generated as module over \( C^\infty(V)^G \). We regard it as a smooth version of a finite type algebra.

Let \( M \) be a finitely generated \( A \)-module. By \cite[Lemma 3]{KNS} it has a resolution \( (A \otimes_{C} V_*, d_*) \) consisting of finitely generated free \( A \)-modules.

**Lemma 2.1.** Assume that (i), (ii) and (iii) or (iii') from Conditions A hold and put
\[
C_n = C^\infty(V)^G \otimes_{\mathcal{O}(\tilde{V})^G} A \otimes_{C} V_n.
\]

Then \( (C_n, \text{id} \otimes d_n) \) is a resolution of the
\[
C^\infty(V)^G \otimes_{\mathcal{O}(\tilde{V})^G} A \text{-module } C^\infty(V)^G \otimes_{\mathcal{O}(\tilde{V})^G} M
\]
by finitely generated free modules. It is split exact as complex of Fréchet spaces.

**Proof.** The exactness of
\[
C_* \to C^\infty(V)^G \otimes_{\mathcal{O}(\tilde{V})^G} M
\]
is a direct consequence of Theorem 1.5. The Fréchet space \( C^\infty(V) \) is isomorphic to a quotient of \( S(\mathbb{Z}) \) \cite[Satz 31.16]{McVo}, and
\[
C^\infty(V) = C^\infty(V)^G \oplus (1 - p_G)C^\infty(V) \quad \text{where} \quad p_G = |G|^{-1} \sum_{g \in G} g.
\]
Hence \( C^\infty(V)^G \) is also isomorphic (as Fréchet space) to a quotient of \( S(\mathbb{Z}) \). Since \( A \) is a finite type \( \mathcal{O}(\tilde{V})^G \)-algebra and \( V_n \) has finite dimension,
\[
C_n = C^\infty(V)^G \otimes_{\mathcal{O}(\tilde{V})^G} A \otimes_{C} V_n
\]
is isomorphic to a quotient of \( S(\mathbb{Z})^d \) for some \( d \in \mathbb{N} \). Recall that \( S(\mathbb{Z})^d \cong S(\mathbb{Z}) \) for any \( d \in \mathbb{N} \). We deduce that \( C_n \) is isomorphic to a quotient of \( S(\mathbb{Z}) \) as well, say via a quotient map
\[
q_n : S(\mathbb{Z}) \to C_n.
\]
By continuity \( \ker(\text{id} \otimes d_n) \) is a closed subspace of \( C_n \). Then \( q_n^{-1}(\ker(\text{id} \otimes d_n)) \) is a closed subspace of \( S(\mathbb{Z}) \). According to [YoWa] Lemma 1.5 there exists a closed linear subspace \( E \) of \( S(\mathbb{Z}) \) such that

\[
S(\mathbb{Z}) = q_n^{-1}(\ker(\text{id} \otimes d_n)) \oplus E.
\]

Dividing out \( \ker(q_n) \), we find

\[
S(\mathbb{Z})/\ker(q_n) \cong \ker(\text{id} \otimes d_n) \oplus E,
\]

another direct sum of Fréchet spaces. By the open mapping theorem \( q_n \) induces a homeomorphism \( S(\mathbb{Z})/\ker(q_n) \rightarrow C_n \), so \( \ker(\text{id} \otimes d_n) \) has the closed complement \( q_n(E) \cong E \). Again by the open mapping theorem, the continuous bijections

\[
(id \otimes d_n)|_{q_n(E)} : q_n(E) \rightarrow \ker(\text{id} \otimes d_{n-1}) \quad n \geq 0
\]

are homeomorphisms. The inverses \( \ker(\text{id} \otimes d_{n-1}) \rightarrow q_n(E) \) provide a splitting of \( (C_n, \text{id} \otimes d_*) \) as differential complex of Fréchet spaces. \( \square \)

We turn to a comparison of the Hochschild homology of \( A \) and of \( C^\infty(V)^G \otimes_{\mathcal{O}(V)^G} A \).

For the finite type algebra \( A \), this can be defined as

\[
HH_n(A) = \text{Tor}_n^{A \otimes A^{\text{op}}}(A, A),
\]

see [Lod] Proposition 1.1.13]. For Fréchet algebras like \( C^\infty(V)^G \otimes_{\mathcal{O}(V)^G} A \), the topology must be taken into account. To that end we regard it as a complete bornological algebra and consider its bornological modules [Mey] §2. For Fréchet modules the bornological structure is equivalent to the topological structure, but there are more bornological modules. The appropriate tensor product is the complete bornological tensor product \( \widehat{\otimes} \), which for Fréchet spaces agrees with the complete projective tensor product [Mey2] Theorem I.87.

The category of bornological modules of a complete bornological algebra \( B \) is made into an exact category by allowing only extensions of \( B \)-modules that are split as extensions of bornological vector spaces. For extensions of Fréchet \( B \)-modules, this just means that they must be split as extensions of Fréchet spaces. It was checked in [Mey] §3 that this is an excellent setting for homological algebra. In particular, a good definition of the Hochschild homology of \( B \) is

\[
(2.1) \quad HH_n(B) = \text{Tor}_n^{B \widehat{\otimes} B^{\text{op}}}(B, B),
\]

in the exact category of bornological \( B \)-modules.

By default we endow all finitely generated \( \mathcal{O}(V)^G \)-modules with the fine bornology [Mey] §2.1, so that complete bornological tensor products also make sense for them. From (2.1) we see that we will have to consider some modules over

\[
C^\infty(V)^G \widehat{\otimes} C^\infty(V)^G \cong C^\infty(V \times V)^G.
\]

Lemma 2.2. Suppose that (i), (ii) and (iii) from Conditions \( A \) hold.

(a) \( \mathcal{O}(V)^G \) is dense in \( C^\infty(V)^G \).

(b) For any finite type \( \mathcal{O}(V)^G \)-algebra \( A \), there is a natural isomorphism of \( C^\infty(V)^G \)-algebras

\[
C^\infty(V)^G \otimes_{\mathcal{O}(V)^G} A \rightarrow C^\infty(V)^G \widehat{\otimes}_{\mathcal{O}(V)^G} A \widehat{\otimes}_{\mathcal{O}(V)^G} C^\infty(V)^G : f \otimes a \mapsto f \otimes a \otimes 1.
\]
Proof. (a) It suffices to show that \( O(\tilde{V}) \) is dense in \( C^\infty(V) \), because from that we can obtain the statement by applying the idempotent \( p_G \). For any \( v \in V \), (iii) yields a natural isomorphism
\[
\tilde{O}(\tilde{V})_v = \hat{FP}_v \cong FP_v = C^\infty(V)_v.
\]
According to [Tou, Corollaire V.1.6] this implies that the closure of \( O(\tilde{V}) \) in \( C^\infty(V) \) is \( C^\infty(V) \).

(b) By Lemma 1.3 (applied to \( \tilde{V} \times \tilde{V} \) with the \( G \times G \)-action),
\[
(C^\infty(V)^G \otimes C^\infty(V)^G)_{O(\tilde{V})^G} \cong (\hat{O}(\tilde{V})^G \otimes \hat{O}(\tilde{V})^G)_{\hat{O}(\tilde{V})^G} \cong C^\infty(V)^G \otimes_{O(\tilde{V})^G} C^\infty(V)^G
\]
is a Fréchet space. Let \( x \in C^\infty(V)^G \otimes A \) and \( f \in C^\infty(V)^G \). By part (a) there exists a sequence \( (f_n)_{n=1}^\infty \) in \( O(\tilde{V})^G \) converging to \( f \). The space \( (2.2) \) is Hausdorff, so limits are unique in there and we can compute
\[
x \otimes f = \lim_{n \to \infty} x \otimes f_n = \lim_{n \to \infty} x f_n \otimes 1 = x f \otimes 1.
\]
Consequently \( (2.2) \) equals
\[
C^\infty(V)^G \hat{\otimes}_{O(\tilde{V})^G} A \otimes_{O(\tilde{V})^G} C^\infty(V)^G = C^\infty(V)^G \hat{\otimes}_{O(\tilde{V})^G} A.
\]
Since \( C^\infty(V)^G \otimes A \) already is Fréchet (by Lemma 1.3), it equals the right hand side of \( (2.3) \). It is easy to see that this isomorphism of Fréchet \( C^\infty(V)^G \)-modules is given by the map in the statement. That map is an algebra homomorphism, so even an isomorphism of \( C^\infty(V)^G \)-algebras. \( \square \)

Lemmas 2.1 and 2.2.b together say that the embedding of bornological algebras \( A \to C^\infty(\tilde{V})^G \otimes_{O(\tilde{V})^G} A \) is isocohomological, in the terminology of [Mey1]. That implies several comparison results for homological properties of the derived module categories of the two involved algebras, see [Mey1, Theorem 35].

One can compute \( HH_n(B) \) (at least when \( B \) is unital) with a completed version of the standard bar-resolution of \( B \) [Loc., §1], but the definition as a derived functor is more flexible. The inclusion \( A \to C^\infty(V)^G \otimes_{O(\tilde{V})^G} A \) induces a chain map between the respective bar-resolutions, and hence induces a natural map
\[
(2.4) \quad HH_n(A) \to HH_n(C^\infty(V)^G \otimes_{O(\tilde{V})^G} A).
\]

Theorem 2.3. Assume that (i), (ii) and (iii) from Conditions A are fulfilled. Then \( (2.4) \) induces a natural isomorphism of Fréchet \( C^\infty(V)^G \)-modules
\[
C^\infty(V)^G \otimes_{O(\tilde{V})^G} HH_n(A) \to HH_n(C^\infty(V)^G \otimes_{O(\tilde{V})^G} A).
\]

Proof. The algebra \( A \otimes A^{op} \) is of finite type over \( O(\tilde{V})^G \otimes O(\tilde{V})^G \). Hence [KNS, Lemma 3] applies to it, and yields a resolution \( (A \otimes A^{op} \otimes V_*, d_*) \) of \( A \) by finitely generated free \( A \otimes A^{op} \)-modules. By definition
\[
(2.5) \quad HH_n(A) = H_n(A \otimes_{A \otimes A^{op}} (A \otimes A^{op}) \otimes V_*, id \otimes d_* = H_n(A \otimes V_*, d_*).
\]
We note that by [KNS, Corollary 1] $HH_n(A)$ is a finitely generated $\mathcal{O}(\tilde{V})^G$-module, so applying $C^\infty(V)^G \otimes_{\mathcal{O}(\tilde{V})^G}$ to it yields a Fréchet $C^\infty(V)^G$-module (Lemma 1.3).

We abbreviate $B = C^\infty(V)^G \otimes A$. By the associativity of $\hat{\otimes}$ ([Mey1], §2.1) there is a natural algebra isomorphism

$$(C^\infty(V)^G \hat{\otimes} C^\infty(V)^G) \otimes_{\mathcal{O}(\tilde{V})^G \otimes \mathcal{O}(\tilde{V})^G} (A \otimes A^{op}) \cong B \otimes B^{op}.$$ \hspace{1cm} (2.6)

Using that we put

$$C_n = (C^\infty(V)^G \hat{\otimes} C^\infty(V)^G) \otimes_{\mathcal{O}(\tilde{V})^G \otimes \mathcal{O}(\tilde{V})^G} (A \otimes A^{op}) \otimes V_n \cong B \otimes B^{op} \otimes V_n.$$ \hspace{1cm} (2.6)

Then Lemma 2.1 says that $(C_*, id \otimes d_*)$ is a free resolution of

in the exact category of complete bornological $B \otimes B^{op}$-modules. By Lemma 2.2 the algebra $B$ is just $B$. Like in (2.5) we obtain

$$HH_n(B) = H_n(B \otimes V, id \otimes d) = H_n(B \otimes V, d).$$ \hspace{1cm} (2.7)

By the exactness of $C^\infty(V)^G \otimes_{\mathcal{O}(\tilde{V})^G}$ there are natural isomorphisms of bornological $C^\infty(V)^G$-modules

$$H_n(B \otimes V, d) \cong H_n(C^\infty(V)^G \otimes A \otimes V, d) \cong C^\infty(V)^G \otimes H_n(A \otimes V, d).$$

Combine that with (2.5) and (2.7). The resulting isomorphism shows that $HH_n(C^\infty(V)^G \otimes A)$ is Hausdorff. It follows that in its construction as

$$H_n(B \otimes V, d) = \ker(d_n)/\operatorname{im}(d_{n+1}),$$

the image of $d_{n+1}$ is closed. Further $\ker(d_n)$ is closed by the continuity of $d_n$, so it is Fréchet, and then dividing out a closed subspace results in another Fréchet space. \hfill $\square$

It would be interesting to draw consequences from Theorem 2.3 for the periodic cyclic homology of $A$ ([Lod], §5.1.3) and of the Fréchet algebra $C^\infty(V)^G \otimes A$ (using the definition from [BrPl] §2 with the complete bornological tensor product). The periodic cyclic homology $HP_n(A)$ of the finite type algebra $A$ was analysed in [KNS] §4. For instance, it follows easily from [KNS, Theorem 10] that $HP_n(A)$ has finite dimension. The inclusion

$$A \to C^\infty(V)^G \otimes A$$

induces a linear map $HP_n(A) \to HP_n(C^\infty(V)^G \otimes A)$.

In some cases this map is a bijection ([Sol1], §1). However, that does not hold in the generality of Theorem 2.3 for instance because $V$ and $\tilde{V}$ can have different cohomology.
3. Modules consisting of differential forms

We preserve the setting of the previous paragraph. To make good use of Theorem 2.3 we will make both

\[ C^\infty(V)^G \otimes_{O(V)^G} HH_n(A) \quad \text{and} \quad HH_n\left( C^\infty(V)^G \otimes_{O(V)^G} A \right) \]

more explicit in some relevant classes of examples. As we are dealing with non-completed tensor products, this involves checking that some modules are finitely generated. There have been ample investigations of the structure of \( C^\infty(V)^G \), starting with [Sch]. On the other hand, \( C^\infty(V) \) has hardly been studied as \( C^\infty(V)^G \)-module.

Let \( \pi \) be a representation of \( G \) on a finite dimensional real vector space \( W \). By classical results of Noether, see for instance [Eis, §13.3], the ring of real valued polynomial functions \( S(W^*) \) on \( W \) is finitely generated as module over \( S(W^*)^G \).

**Theorem 3.1.** Let \( G \) be a finite group.

(a) \( C^\infty(W) \) is generated as \( C^\infty(W)^G \)-module by a finite subset of \( S(W^*) \).

(b) Let \( V \) be a smooth manifold with a smooth \( G \)-action. Then \( C^\infty(V) \) is finitely generated as \( C^\infty(V)^G \)-module.

**Proof.** (a) This is contained in [Poc, Lemme III.1.4.1], but in disguise. Namely, it is stated there that, for any finite dimensional real \( G \)-representation \( (\pi', W') \),

\[ C^\infty_G(W, W') = \{ f \in C^\infty(W, W') : f(\pi(g)w) = \pi'(g)f(w) \forall g \in G, w \in W \} \]

is a finitely generated \( C^\infty(W)^G \)-module. We claim that, for \( W' = C[G] \) the left regular representation, there is an isomorphism of \( C^\infty(W)^G \)-modules

\[
\begin{align*}
C^\infty(W) & \leftrightarrow C^\infty_G(W, C[G]), \\
\phi_1 & \leftrightarrow [w \mapsto \sum_{g \in G} f(\pi(g^{-1})w)g] . \\
\end{align*}
\]

(3.1)

Indeed, the equivariance condition \( \phi(\pi(g)w) = g\phi(w) \) means precisely that

\[ \phi_g(w) = \phi_1(\pi(g^{-1})w) \text{ for all } w \in W. \]

Hence the two maps in (3.1) are mutually inverse. The proof of [Poc, Lemme III.1.4.1] uses only polynomial functions on \( W \otimes W^* \) as generators, so via the isomorphism (3.1) we can conclude that \( C^\infty(W) \) is generated by a finite subset of \( S(W^*) \). In fact any set that generates \( S(W^*) \) as \( S(W^*)^G \)-module will do.

(b) By [Mos, Theorem 6.1], \( V \) can be embedded \( G \)-equivariantly as a closed submanifold in a space \( W \) as in part (a). Thus we may and do regard \( V \) as a subspace of \( W \). With part (a) we choose a finite set of generators \( \{ f_i \} \) for \( C^\infty(W) \) as \( C^\infty(W)^G \)-module. According to [Tou, Théorème IX.4.3], the restriction map

\[ C^\infty(W) \to C^\infty(V) : f \mapsto f|_V \]

is surjective. Hence the functions \( f_i|_V \) generate \( C^\infty(V) \) as \( C^\infty(V)^G \)-module. \( \square \)

In the algebraic setting, a theorem of Serre says that \( \Omega^n(\tilde{V}) \) is finitely generated as \( O(\tilde{V}) \)-module, and hence also as \( O(V)^G \)-module. Similarly, the smooth Serre–Swan theorem says that \( \Omega^n_{sm}(V) \) is finitely generated as \( C^\infty(V) \)-module, for any \( n \in \mathbb{Z}_{\geq 0} \). By Theorem 3.1 it also finitely generated as \( C^\infty(V)^G \)-module.
In view of the Hochschild–Kostant–Rosenberg theorem \([\text{Lod}]\) Theorem 3.4.4, the Hochschild homology of finite type algebras will involve differential forms on varieties related to \(\tilde{V}\). We will study such modules in a setting that starts with (i) and (ii) from Conditions [A]. We assume that an embedding \(1 \to DAVID KAZHDAN AND MAARTEN SOLLEVELD\)

Let \(Y\) be a finite disjoint union of complex affine varieties \(\tilde{Y}_j (j \in J)\), not necessarily of the same dimension, each of which has the same properties as those of \(\tilde{Y}_1\) just listed. Let \(Y\) be the disjoint union of the \(Y_j\).

Let us point out that the standard and most instructive case of the next three results is simply \(\tilde{Y} = \tilde{V}, Y = V\).

**Lemma 3.2.** With the above assumptions, let \(C^\infty(V)^G\) act on \(\Omega^n_{\text{sm}}(Y)\) via \(i^*\).

(a) \(\Omega^n(\tilde{Y})\) is finitely generated as \(\mathcal{O}(\tilde{V})^G\)-module.

(b) \(\Omega^n_{\text{sm}}(Y)\) is generated as \(C^\infty(V)^G\)-module by a finite subset of \(\Omega^n(\tilde{Y})\).

**Proof.** (a) By assumption \(i(\tilde{Y})\) is closed in \(\tilde{V}\), so the restriction map \(\mathcal{O}(\tilde{V}) \to \mathcal{O}(i(\tilde{Y}))\) is surjective. As \(i|_{\tilde{V}}\) is an isomorphism \(i^* : \mathcal{O}(\tilde{V}) \to \mathcal{O}(\tilde{Y})\) is surjective. In particular \(\Omega^n(\tilde{Y})\) is a finitely generated module, over \(\mathcal{O}(\tilde{V})\) as well as over \(\mathcal{O}(\tilde{Y})\). Since \(\mathcal{O}(\tilde{V})\) is the integral closure of \(\mathcal{O}(\tilde{V})^G\) in the quotient field of \(\mathcal{O}(\tilde{V})\), it has finite rank over \(\mathcal{O}(\tilde{V})^G\) \([\text{Eis}]\) Proposition 13.14]. Hence \(\Omega^n(\tilde{Y})\) is also finitely generated as \(\mathcal{O}(\tilde{V})^G\)-module.

(b) By the smooth Serre–Swan theorem, \(\Omega^n_{\text{sm}}(Y_j)\) is finitely generated over \(C^\infty(V_j)\). As \(i(Y_j)\) is a closed submanifold of \(V\), the restriction map \(C^\infty(V) \to C^\infty(i(Y_j))\) is surjective \([\text{Tou} \text{Théorème IX.4.3}]. Since \(i|_{Y_j}\) is a diffeomorphism, also

\[(3.2) \quad i^* : C^\infty(V) \to C^\infty(Y)\]

is surjective.

In particular \(\Omega^n_{\text{sm}}(Y_j)\) is a finitely generated \(C^\infty(V)\)-module, and so is \(\Omega^n_{\text{sm}}(Y) = \bigoplus_{j \in J} \Omega^n(Y_j)\). From the definition of the module structures we see that the tensor products

\[C^\infty(V)^G \otimes \Omega^n(\tilde{Y}), \quad C^\infty(V)^G \otimes \mathcal{O}(\tilde{Y}) \otimes \Omega^n(\tilde{Y}), \quad C^\infty(V)^G \otimes \mathcal{O}(\tilde{V}) \otimes \Omega^n(\tilde{Y}).\]

have the same image in \(\Omega^n_{\text{sm}}(Y)\), under the natural action maps. By Theorem [3.1] b and (3.2) the last one has the same image as

\[C^\infty(V) \otimes \Omega^n(\tilde{Y}) \quad \text{and} \quad C^\infty(V) \otimes \mathcal{O}(\tilde{Y}) \otimes \Omega^n(\tilde{Y}).\]

The latter equals \(\Omega^n_{\text{sm}}(Y)\), so \(\Omega^n(\tilde{Y})\) generates \(\Omega^n_{\text{sm}}(Y)\) as \(C^\infty(V)^G\)-module. By part (a) that can be achieved with a finite subset of \(\Omega^n(\tilde{Y})\). \(\square\)

Consider a \(\mathcal{O}(\tilde{V})^G\)-submodule \(M\) of \(\Omega^n(\tilde{Y})\), where the action goes via \(i^*\). Although it might seem obvious that \(C^\infty(V)^G \otimes \mathcal{O}(\tilde{V})^G \otimes M\) embeds in \(\Omega^n_{\text{sm}}(Y)\), that is actually about as difficult as Theorem [1.5].
Proposition 3.3. Assume that (i), (ii) and (iii) from Conditions $A$ hold and let $M,Y$ and $\tilde{Y}$ be as above. The natural homomorphism of Fréchet $C^\infty(V)^G$-modules
\[
C^\infty(V)^G \otimes_{\mathcal{O}(V)^G} M \to \Omega_{sm}^n(Y)
\]
is injective.

*Proof.* By Theorem 1.5 the natural map
\[
C^\infty(V)^G \otimes_{\mathcal{O}(V)^G} M \to C^\infty(V)^G \otimes_{\mathcal{O}(V)^G} \Omega^n(\tilde{Y})
\]
is injective. Therefore we may assume that $M = \Omega^n(\tilde{Y})$. Then the statement factors naturally as a direct sum indexed by $j \in J$. It suffices to consider one such direct summand, say
\[
(3.3)
\]
The formal completion of $\Omega_{sm}^n(Y_1)$ as $C^\infty(V)^G$-module at $Gv \in V/G$ is
\[
\bigoplus_{y \in \iota^{-1}(Gv)} FP_{Gv}^{\text{sm}} \otimes_{C^\infty(V)^G} C^\infty(Y_1)_y \wedge^n (T_y(Y_1)^*) = \bigoplus_{y \in \iota^{-1}(Gv)} C^\infty(Y_1)_y \wedge^n (T_y(Y_1)^*).
\]
Using assumption (iii) we can also compute the formal completion of the left hand side of (3.3):
\[
\left(C^\infty(V)^G \otimes_{\mathcal{O}(V)^G} \Omega^n(\tilde{Y})\right)_{Gv}^\wedge = \bigoplus_{y \in \iota^{-1}(Gv)} FP_{Gv}^{\text{sm}} \otimes_{C^\infty(V)^G} \Omega_{sm}^n(Y_1)_y \wedge^n (T_y(\tilde{Y})^*)
\]
\[
= \bigoplus_{y \in \iota^{-1}(Gv)} \Omega_{sm}^n(Y_1)_y \wedge^n (T_y(\tilde{Y})^*).
\]
Assumption (iii) and the construction of $Y_1$ imply that $T_y(\tilde{Y}) = T_y(Y_1) \otimes_{\mathbb{R}} \mathbb{C}$. From that and the above we see that the map
\[
\left(C^\infty(V)^G \otimes_{\mathcal{O}(V)^G} \Omega^n(\tilde{Y})\right)_{Gv}^\wedge \to \Omega_{sm}^n(Y_1)_Gv
\]
induced by (3.3) is injective. Now the same argument as for $\mu$ in the proof of Theorem 1.5 shows that (3.3) is injective. \qed

Describing the image of the map from Proposition 3.3 is another issue. One would like to think of it as some closure of $M$ in $\Omega_{sm}^n(Y)$, but it in general it is not clear whether the image is closed. To overcome that, we specialize to submodules of $\Omega^n(\tilde{Y})$ that are direct summands. Let $p$ be an idempotent in the ring of continuous $C^\infty(V)^G$-linear endomorphisms of $\Omega_{sm}^n(Y)$, such that $p$ stabilizes $\Omega^n(\tilde{Y})$. Then
\[
(3.4)
\]

\[
\Omega_{sm}^n(Y) = p\Omega_{sm}^n(Y) \oplus (1-p)\Omega_{sm}^n(Y),
\]
so $p\Omega_{sm}^n(Y)$ is a closed $C^\infty(V)^G$-submodule of $\Omega_{sm}^n(Y)$. Similarly $p\Omega^n(\tilde{Y})$ is an $\mathcal{O}(\tilde{V})^G$-submodule and a direct summand of $\Omega^n(\tilde{Y})$.

Lemma 3.4. Assume (i), (ii) and (iii) from Conditions $A$. The natural map
\[
\mu : C^\infty(V)^G \otimes_{\mathcal{O}(V)^G} p\Omega^n(\tilde{Y}) \to p\Omega_{sm}^n(Y)
\]
is an isomorphism of Fréchet $C^\infty(V)^G$-modules.
Proof. By construction the image of $\mu$ is contained in $p\Omega^p_{\text{sm}}(Y)$ and we know from Proposition 3.3 that $\mu$ is injective. By Lemma 3.2.b any $m \in p\Omega^p_{\text{sm}}(Y)$ can be written as a finite sum $m = \sum_i f_i \omega_i$ with $f_i \in C^\infty(V)^G$ and $\omega_i \in \Omega^n(\tilde{Y})$. We compute

$$m = p(m) = p\left(\sum_i f_i \omega_i\right) = \sum_i f_i p(\omega_i) \in \mu\left(C^\infty(V)^G \otimes \Omega^n(\tilde{Y})\right).$$

In other words, $\mu$ is surjective. In view of Proposition 3.3 $\mu$ is a continuous bijection between Fréchet spaces. Now the open mapping theorem says that it is a homeomorphism. □

References


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