Open Problems for 2006 Hanoi Conference Proceedings

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Let $R = k^{[n]}$, the polynomial ring in n variables over a field k. Let $\mathrm{GA}_2(R)$ denote the automorphisms of $\mathbb{A}^2_R$.

Problem 1. *Are all elements of $GA_2(R)$ stably tame?*

Remark. The *length* of an element of $\mathrm{GA}_2(R)$ is defined the minimal number of elementary automorphisms in a factorization of it in $\mathrm{GA}_2(K)$, where K is the field of fractions of R. This question is answered affirmatively for elements of length ≤ 3 in [1]. Sooraj Kuttykrishnan has now resolved the length 4 case. These results assume only that R be a UFD, with Kutttykrishnan's result requiring a further mild condition.

Problem 2. *What is the structure of $GA_2(R)$?*

Remark. Actually it is proved in [2] and [3] that $\mathrm{GA}_2(R)$ has the structure of an amalgamated free product

$$\mathrm{Af}_2(k) *_{\mathrm{Bf}_2(k)} W$$

Where $\mathrm{Af}_2(k)$ is the affine group over k, $\mathrm{Bf}_2(k)$ is the lower triangular affine group, and W is an obscure group which is a bit difficult to define (see Theorem 1 of [2]). We would like to have a better understanding of W.

References

[1] E. Edo, Totally stably tame variables, J. Algebra 287 (2005) 15-31.

[2] D. Wright, The amalgamated free product structure of $\mathrm{GL}_2(k[X,Y])$ and the weak Jacobian theorem for two variables, J. of Pure and Applied Algebra 12, (1978), 235-251.

[3] D. Wright, Normal forms and the Jacobian conjecture, Automorphisms of Affine Spaces (A. van den Essen, ed.), Kluwer Academic Publishers, The Netherlands, (1995), 145-156.

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