Algebra III (Algebraic Geometry II) WS15-16
Fachbereich Mathematik und Informatik
J-Prof. Victoria Hoskins
Teaching Assistant: Eva Martínez

Algebraic Geometry II

Exercise Sheet 15 : Revision Sheet

Hand-in date: 10am, Monday 9th February.

Throughout this sheet k is an algebraically closed field (of arbitrary characteristic) and by a scheme, we mean a finite type scheme over $\operatorname{Spec} k$.

For each of the following statements, answer true or false and justify your answer by giving an outline of the proof or by providing a (counter)example.

a) The moduli problem of classifying 1-dimensional linear subspaces of k^n has a fine moduli space.

b) The moduli problem of classifying isomorphism classes of vector bundles of fixed rank and degree on a smooth projective curve has a coarse moduli space.

c) A moduli problem that has a family with the jump phenomenon can have a coarse moduli space.

d) There exist non-reduced affine algebraic group schemes.

e) The additive group $\mathbb{G}_a$ is linearly reductive.

f) Any torus is linearly reductive.

g) A smooth affine algebraic group scheme is linearly reductive if and only if it is reductive.

h) For an affine algebraic group G acting on a scheme X, the closure of every orbit contains a unique closed orbit.

i) Every linear representation of a finite group over an algebraically closed field of characteristic zero admits a Reynolds operator.

j) A categorical quotient of an affine algebraic group acting on an irreducible scheme is irreducible.

k) A GIT quotient of a reductive group acting on a quasi-projective variety is also a quasi-projective variety.

l) If an affine algebraic group acts on an affine scheme with finitely generated invariant ring, then the spectrum of this invariant ring gives a good quotient of the action.

m) For a reductive group acting linearly on a projective scheme, the orbit of every stable k-point is closed in the semistable locus.

n) Every line bundle $\mathcal{O}_{\mathbb{P}^n}(r)$ admits a linearisation of the natural PGL_{n+1}-action on $\mathbb{P}^n$.

o) The natural SL_n-action on $\mathbb{A}^n$ given by left multiplication admits only one linearisation.

p) For the moduli problem of hypersurfaces of fixed degree d in $\mathbb{P}^n$ up to projective equivalence, there is a family with the local universal property.

q) Every smooth hypersurface in $\mathbb{P}^n$ is semistable.

r) There is a coarse moduli space for semistable hypersurfaces of a fixed degree d in $\mathbb{P}^1$ up to projective equivalence.

s) A cuspidal plane cubic curve is not semistable.