The International Conference on Diophantine Analysis and its Applications in Honour of Acad. Sprindžuk (1936-1987).

September 1-8, 1996
Minsk (Belarus)

ABSTRACTS

The International Conference on Diophantine Analysis and its applications

V. Zudilin

Russia

Minsk 1996

Sponsors:

- Institute of Mathematics of the Belarus Academy of Sciences
- The International Mathematical Union
- Belarus Department of Soros Foundation
- Belarus Agrarian Technical University

With best regards
D. Bertrand
Alf
Pierre Debes
Rob Tijdeman
Kunrui Yu
Dubickas
Keijo Väänä
andrew Pollington
Attila Pintér
Hans Peter Schlickewei
Maurice Dodson
Sanju Velani
Максим Скриганов
Detta Dickinson
(Yuan Jin)
Michel WALDSCHMIDT
Sinnou DAVID

List of participants:

G. Arhipov
V. Beresnevich
V. Bernik
D. Bertrand
V. Borbat
S. David
P. Debes
H. Dickinson
M. Dodson
A. Dubickas
E. Gaigalas
A. Galochkin
G. Gusev
L. Gutnik
V. Ilkiv
Y. Jin
N. Kalosha
A. Karatsuba
L. Kogan
O. Koroleva
S. Kotov
E. Kovalevskaya
J. Kubilius
M. Kudrjavtsev
A. Laurincikas
E. Matveev
D. Mitkin
B. Moroz
I. Morzova
N. Moshchevitin
R. Nair
Yu. Nesterenko
P. Philippon
A. Pinter
A. Pollington
A. van der Poorten
B. Ptashnyk
Romanovsky
H. Sakovich
V. Salihov
H.P. Schlickewei
M. Skriganov
S. Stepanov
R. Tijdeman
K. Tishchenko
K. Väänänen
D. Vasilyev
S. Velani
T. Vepkhvadze
M. Waldschmidt
K. Yu
V. Zudilin

Vladimir Gennadievich Sprindžuk
(22.07.1936 – 26.07.1987)

V.G Sprindžuk was a famous authority on the theory of Diophantine equations, Diophantine approximation and transcendental Number Theory. An alumnus of the Belorussian State University (1954-1959, where he was an undergraduate) and of the State University of Vilnius (1959-1962, where he undertook his postgraduate studies), he obtained his PhD in 1963, and his DSc degree in 1965. In 1969 he was made a full professor and a member of the Editorial Board of the Vesti of the Akademija Nauk BSSR (Mathematics). The following year he joined the Editorial Board of Acta Arithmetica, and in 1986 Prof. Sprindžuk became an Academician of the Belorussian Academy of Sciences.

While a pupil of the 8-th form, Volodya Sprindžuk read the "Three pearls of Number Theory" by A.Ya. Khintchin and the "Basic Number Theory" by I.M. Vinogradov and decided to become a professional mathematician. As a first year undergraduate student, V. Sprindžuk solved a problem posed by A.Ya. Khintchin, published it as his first paper, and wrote to the author of the problem. The four/five letters from Professor Khintchin were a source of great delight and inspiration for the young man. V. Sprindžuk read his first paper at a conference in Vilnius. The rector of the University of Vilnius Professor J. Kubilius attended his lecture and appreciated his result, later Prof. J. Kubilius became a scientific advisor of G. Sprindžuk. Both Professor Kubilius and Professor Yu.V. Linnik greatly influenced the studies of young Sprindžuk. In particular, Yu.V. Linnik directed Sprindžuk's efforts towards the theory of Diophantine approximation and transcendental number theory, an area of mathematics which lay outside his own interests. Linnik's judgement about the young mathematical talent proved to be right. During his postgraduate studies V. Sprindžuk became interested in the metric theory of transcendental numbers his PhD thesis was entitled "Metric theorems on Diophantine approximations by algebraic number of bounded degree". A year later V. Sprindžuk proved the Mahler conjecture in the theory of transcendental numbers. In the course of this work he developed a new method, nowadays called the method of essential and non-essential regions. In 1965 V.G. Sprindžuk defended his DSc dissertation "The Mahler problem in the metric theory of numbers" at the Leningrad State University. His dissertation and the papers of W. Schmidt of 1964 may be regarded as groundwork for a new area of

research, the metric theory of Diophantine approximation of dependent variables. Sprindžuk's further studies in this direction are widely known.

In 1969, V. Sprindžuk was elected a corresponding member of the Belorussian Academy of Sciences, became a professor and the head of the department of number theory at the Mathematical Institute of the Belorussian Academy of Sciences. He started lecturing at the University of Minsk.

In the late sixties V. Sprindžuk began studying the theory of transcendental numbers and Diophantine equations. In 1969-71 he investigated the arithmetic properties of the Siegel hypergeometric E-functions with algebraic parameters and defined a wider class of E^*-functions. His detailed studies of the Thue equation in algebraic number fields proved to be useful for the effective solution of a wide class of Diophantine equations and allowed him to study the possibility of effective approximations to algebraic numbers both in archimedean and non-archimedean domains. Sprindžuk's results are based on the connections between linear forms of logarithms in different norms. He observed that if a linear form is p-adically "not too small" then it cannot be too small in any other norm, be it archimedean or non-archimedean. A quantitative variant of this criterion led Sprindžuk to several effective results concerning the representation of numbers by binary forms, estimates for the magnitude of maximal prime factor of a binary form and the rational approximations to algebraic integers. He discovered in particular, a relation between the magnitude of the solutions of Diophantine equations and the number of classes of ideals, as well as some constructions of algebraic fields with the large class number.

These and other results of his are widely known. Professor Sprindžuk was an invited speaker at the ICM-1970 in Nice; he lectured at numerous international conferences in 1973, delivered lectures as a guest of Polish Academy of Sciences; in 1975 and 1977 he was a visiting professor of the Slovakian Academy of Sciences, in 1980 he visited the University of Paris.

In late seventies V. Sprindžuk began developing a theory of arithmetical specializations in polynomials and in the algebraic number fields. He investigated the relations between the coefficients in the power series representations of algebraic functions and the approximation properties of their special values. His theory allowed a universal Hilbert set to be made effective.

Professor Sprindžuk is an author of 75 papers, two thirds of which are translated into English and published in international editions. He wrote three monographs: Mahler's problem in metric number theory, Minsk, 1967 (AMS translation, 1969); Metric theory of Diophantine approximations, Moscow, 1977 (translation: Washington D.C, 1979); Classical Diophantine Equations, Moscow, 1982 (translation: Springer Lecture notes in Mathematics, 1559 (1993), Springer-Verlag).

Sprindžuk's methods and ideas proved to be an inspiration to many mathematicians. He devoted A lot of time and effort to the education of young mathematicians. He worked as chairman at the mathematical olympiads of the Republic of Belarus and lectured at the University of Minsk. Nine of his pupils obtained PhD, and one of them is a Doctor of Science. In 1986 V.G. Sprindžuk became a member of the Belorussian Academy of Sciences.

Professor Sprindžuk died on 26 July 1987. He will be remembered by mathematicians for his work, by his students for his inspiring teaching and by the institute of Mathematics for his Laboratory of Number Theory.

ABSTRACTS

Additive problems of Varing type in multidimensional vector space

G.I. Arhipov, V.N. Chubarikov

MIRAN im. Steklova, ul. Vavilova 42, Moscow, Russia, phone 9496696

We proved the theorems on the values $G_j(n)$, $j = 0, 1, \ldots, 5$, analogous to the well known Hardy function in the Varing problem, which correspond to the Hilbert-Kamke problem in natural and prime numbers, Varing problem for integer polynomials, multidimensional Hilbert-Kamke problem, Hilbert-Kamke problem in Gaussian integers. The quantities $G_j(n)$ behave essentially differently for different j, $0 \leq j \leq 5$, with n tending to infinity. The results were obtained jointly with V.N. Chubarikov.

The best approximation of points of smooth three dimmentional curves

V. V. Beresnevich

Institute of Mathematics, Academy of Sciences of Belarus, 220072, Surganova 11, Minsk, Belarus
e-mail: *imanb%imanb.belpak.minsk.by@demos.su*

There was a great interest in the diophantine approximation of curve points since works of Mahler, Kubilius, Sprindzuk and Scmidt. For about 20 years the only studied general case of the extremality of curves was that of plane curves. It has been investigated by W.Schmidt [1]. We deal with smooth three dimensional curves with torsion vanishing almost nowhere. Their extremality has been proven in [2]. We consider finesses of the method used and its improvement which gives a Khinchine–type version of the result for the convergence case.

References:

1. *W.M.Schmidt.* Metrische Sätze über simultane Approximation abhängiger Grössen, Monatsh. Math. 1964, Bd.63, N 2, pp.154–166.
2. *Beresnevich V.V., Bernik V.I.*, On a metrical theorem of W. Schmidt // Acta Arithmetica, LXXV.3(1996), p.219–233.

The rational points near the smooth surfaces and the transference principle of Khintchin

V. I. Bernik
Surganova 11, Institute of Mathematics, Minsk, Belarus.
e-mail: imanb%imanb.belpak.minsk.by@demos.su

It is well known that in the theory of diophantine approximations an upper estimate for the number of rational points near curves and surfaces leads to the essential theorems (Kubilius, Cassels, Sprindžuk). However, there are many results which can be obtained with other methods. We show how the transference principle of Khinchin allows to obtain the information on the distribution of rational points near smooth surfaces.

Transcendence properties of algebraically reducible hypergeometric equations

D. Bertrand

it Univ Paris IV, Mathematiques, case 247, 4, Place Jussieu F-75252 Paris Cedex 05
e-mail: bertrand@mathp6.jussieu.fr

The work of Salikhov and of Brownawell-Beukers-Heckman provides a thorough description of the set of algebraically irreducible confluent hypergeometric equations. We here report on the recent results of K. Boussel and of E.Compoint, computing the orbits of the differential Galois group in the case of linearly reducible [1] and of quadratically reducible [2] equations.

As an illustration of the scope of their methods, consider the following hypergeometric E-function (corresponding to the exceptional group G_2):

$$f_b(z) = {}_1F_6(1/2; 1+b, 1+b, 1-b, 1-b, 1, 1, 1; (z/6)^6),$$

where $2b$ is a rational number not in $\mathbf{Z}$, and let $F_b(z) = \int_0^z f(t)\,dt$. The seven functions $f_b(z)$, $f_b'(z)$,..., $f_b^{(6)}(z)$ are linearly independent, but homogeneously algebraically dependent, over $\mathbf{C}(z)$. On combining [2], [1], Kolchin's theorem on subgroups of products, and Shidlovskii's "Third fundamental theorem", one can derive :

Theorem. *Let a_1, ..., a_n be non zero algebraic numbers, such that none of the (a_i/a_j) $(i \neq j)$ is a 6-th root of unity. Then the values at a_1, ..., a_n of f_b (resp. F_b) and of all its derivatives generate a field of transcendence degree $6n$ (resp. $7n$) over $\mathbf{Q}$.*

References:

1 K. Boussel: Opérateurs hypergéométriques résolubles: décompositions et groupes de Galois différentiels; Ann. Fac. Sc. Toulouse, 5, 1996 (to appear).

2 E. Compoint: Equations différentielles, relations algébriques et invariants; These Univ. Paris VI, Juin 1996.

On the complex variant of Baker's conjecture.

V.N Borbat
Mogilev State Pedagogical Institute, Mogilev

According to the conjecture of A. Baker for any $\varepsilon > 0$ the inequality

$$|a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0| < (\textstyle\prod_{i=1}^{n} |a_i| + 1)^{-1-\varepsilon}$$

has only finite number of solutions in integer vectors $\overline{a} = (a_0, a_1, \ldots, a_n)$ for almost all x. The author and V.I Bernik have proven this conjecture for $n < 4$.

Theorem: *In complex case the conjecture of Baker is true for $n \leq 6$ with the exponent $-1/2 - \varepsilon$ in the right-hand part of the inequality.*

Arithmetic of abelian varieties

S. David

Universite de Paris 6, Dpt. Math., VM129994,
Jussieu 75005 Paris, France.
e-mail: david@mathp6.jussieu.fr

We describe the recent results obtained towards the generalized Bogomolov conjecture on the density of algebraic points of small height of subvarieties of abelian varieties, and the links with the normalized height of the variety.

We also starting with the multiplicative analogue discuss of the various quantitative aspects which are either known or can be conjectured.

An application of Diophantine Approximation to pseudo–differential operators

H. Dickinson

Department of mathematics, University of York, UK
e–mail: hd3@unix.york.ac.uk

Some exceptional sets which apear when the hypoellipticity and normal forms of pseudo–differential operators are shown. These sets are very well approximable and so of zero Hausdorff dimension. The concept of logarithmic Hausdorff dimension is introduced and then obtained for certain sets via a theorem on Hausdorff measure and linear forms.

The maximal modulus of a non-reciprocal algebraic integer

A. Dubickas

Dept. Math., Vilnius University, Naugarduko 24,
Vilnius 2006, Lithuania e-mail: arturas.dubickas@mat.vu.lt

An algebraic integer α of degree $n \geq 2$ is called reciprocal if its minimal polynomial satisfies the identity $P(x) \equiv x^n P(1/x)$. Denote by $|\overline{\alpha}|$ the maximal modulus of conjugates of α

Theorem. *Suppose that α is a non-reciprocal algebraic integer of degree n. Then for all sufficiently large n*

$$|\overline{\alpha}| > 1 + 0.3056/n$$

On the other hand, for the polynomial $x^n + x^{2n/3} - 1$, where $3|n$, we have

$$|\overline{\alpha}| = 1 + \frac{0.4217\ldots}{n}$$

.

Zeta-functions of binary Hermitian forms

E. Gaigalas

Vilnius, Lithuania e-mail: Edmundas.Gaigalas@mat.vu.lt

Let $\widetilde{H}(\Delta)$ be the set of orbits of positive definite binary hermitian quadratic forms of discriminant Δ of the relatively imaginary quadratic

extension $Q(i,\sqrt{2}) \supset Q(\sqrt{2})$, where B is the ring of integers of the field $Q(i,\sqrt{2})$, $\mathrm{St}(f)$ is the stabilizer of the form f, $[f]$ is the orbit of f and

$$Z(\Delta,s) = \sum_{[f]\in\widetilde{H}(\Delta)} \frac{1}{\mathrm{card}\ \mathrm{St}(f)} \sum_{(u,v)\in B\times B} N_{Q(\sqrt{2})/Q}(f(u,v))^{-1-s}$$

is the corresponding zeta-function.

Then $Z(\Delta,s)$ has the Euler product expansion in $\mathrm{Re}\, s > 1$:

$$Z(\Delta,s) = \Theta(\Delta,s)\,\zeta_{Q(\sqrt{2})}(s)\,L(s+1,-\frac{1}{p})^{-1}L(s+1,-\frac{2}{p})^{-1}$$

where $L(s,\chi)$ is the Dirichlet L–series, $\zeta_{Q(\sqrt{2})}(s)$ is Dedekind zeta-function,

$$\Theta(\Delta,s) = (1-(1+(-1)^t)2^{-s-1}-(1+(-1)^{t-1})2^{-s(t+2)-1})(1-2^{-2s})^{-1}\times$$

$$\times \prod_{\substack{p\equiv\pm1\,(8)\\ p^t||N(\Delta)}} \frac{(1-(\frac{-1}{p})p^{-s(t+1)})^2}{(1-p^{-s})^2} \prod_{\substack{p\equiv\pm3\,(8)\\ p^t||N(\Delta)}} \frac{1-p^{-2s(t+1)}}{1-p^{-2s}}.$$

A new proof of Gelfond–Schneider's theorem

A. I. Galochkin

Russia, Moscow State University
E-mail: gal@nw.math.msu.su

The talk presents the new proof of the classical theorem by A.O. Gelfond and Th. Schneider (1934).

Theorem. *Let* $\alpha \in \overline{\mathbb{Q}}$, $\alpha \notin \mathbb{Q}$, $a \in \overline{\mathbb{Q}}$, $\alpha \log a \neq 0$. Then $a^\alpha \notin \overline{\mathbb{Q}}$.

The proof is based on the construction of the auxilary function

$$F(z) = \sum_{k,l=0}^{n-1} A_{kl} e^{(k+l\alpha)z}, \qquad A_{kl} \in \mathbb{Z}_{\mathbb{Q}(\alpha)},$$

with large order of zero only at the point $z = 0$.

We do not use the process of interpolation but we use some upper estimate of the order of zeroes of the function $F(z)$.

On Isometric method in Number Theory and Diophantine Analysis

G.I. Gusev

Russia, 410005, Saratov, Astrakhanskaya, 148, 12

Suppose K is a locally compact non-archimedean field, π is its prime element, V is the ring of integers of K and E is the group of units of K. Then on V^n one defines a group of standard isometries

$$\sigma : X^T \to UX^T + \pi(f_1(X), \ldots, f_n(X))^T,$$

where $U = (u_{ij})_{n\times n}$, $((u_{ij}) \in V, \det U \in E)$ is the unimodular matrix of the dimension n,

$$f_i(X) = \sum_{\alpha_1\geq 0} \cdots \sum_{\alpha_n \geq 0} c^{(i)}_{\alpha_1\cdots\alpha_n} X_1^{\alpha_1}\cdots X_n^{\alpha_n},$$

$1 \leq i \leq n$ is a power series, with coefficients satisfying

$$|c^{(i)}_{\overline{\alpha}}| \leq 1, \quad \lim_{|\overline{\alpha}|\to\infty} c^{(i)}_{\overline{\alpha}} = 0, \quad (|\overline{\alpha}| = \sum_{k=1}^{n} \alpha_k)$$

It is well known, that for any matrix $A = (a_{ij})_{n\times n}$ with elements from field K there is at least one such pair of unimodular matrices (U_1, U_2) of size n that $D = U_1 A U_2$ is a diagonal matrix:

$$D = [\pi^{\nu_1}, \ldots, \pi^{\nu_r}, 0, \ldots, 0],\ (\nu_1 \leq \nu_2 \ldots \leq \nu_r).$$

Here r is the rank of matrix A_i and ν_i, $1 \leq i \leq r$ are uniquily defined integers which we call the exponential invariants of A.

The group of isometries shown above allows partitioning into the classes of equivalence for the analytical functions of n variables $\overline{F}(X) = (F_1(X), \ldots, F_n(X))$, where $F_i(X) = \sum f^{(i)}_{\overline{\alpha}} X_1^{\alpha_1} \ldots X_n^{\alpha_n}$ is a power series with coefficients from K converging in some area around zero. From each class one selects the function of simplest kind called a canonic form of this class.

If the matrix of Jacoby $M_{X_0} J(\overline{F})$ for the vector-function $\overline{F}$ at $X_0 \in V^n$ is non-degenerate then for some n-dimensional π-adic ball $S_n(\overline{0}, R)$ we have isometrical equivalence

$$\overline{F}(X)^T \simeq \overline{F}(X_0)^T + M_{X_0} J(\overline{F})(X - X_0)^T$$

The transition from the initial vector-function to its canonical form very often facilitates the solution of some problems in Number Theory and Diophantine Analysis.

Systems of linear forms such that their coefficients are $\mathbb{Q}$-proportional to the $ln(2)$ and values of Zeta-function of Riemann.

L. A. Gutnik

Moscow State Institute of Electronics and Mathematics

Review of papers

[1] L. A. Gutnik. To the question of smallness of some linear forms. Moscow,1993,VINITI.

[2] - Systems of vectors such that their coordinates are linear combinations with algebraic coefficients of logarithms of algebraic numbers. Moscow, 1995, VINITI

[3] - Linear forms such that their coefficients are $\mathbb{Q}$-proportional to the $ln(2)$ and of $\zeta(s)$. Moscow, 1996, VINITI.

Viniti is an abbreviation of All-Russian Institute for Scientific and Technical Information.

Small denominators of multipoint problem for partial differential system

V.S. Il'kiv, B.O, Salyga

290049, Lvov, Vernadskogo 36/33, Ukrain

We consider the multipoint problem for the system of partial differential equations

$$(\partial/\partial t)^n u(t,x) = L(D)v(t,x), \quad t \in [\gamma_1, \gamma_2], \tag{1}$$

$$\alpha(D)v(t_j, x) = \phi_j(x), \quad t_j = t_1 + (j-1)\Delta t, \quad j = \overline{1, np} \tag{2}$$

where $u(t,x)$ is the p-vector to be found, $\phi_j(x)$ are known functions, $v(t,x) = col(u, \partial u/\partial t, \ldots, (\partial/\partial t)^{n-1} u))$, $L(D) = (L_n(D), \ldots, L_1(D))$, $\alpha(D) = (\alpha_n(D), \ldots, \alpha_1(D), \alpha_0(D))$, L_j are square j-degree polynomial p-matrices, α_j are j-degree polynomial p-row, $\gamma_1 \leq t_1 < \ldots < t_{np} \leq \gamma_2$, $x = (x_1, \ldots, x_m)$.

We shall seek a solution of the problem (1), (2) in Hilbert space of x-periodical functions.

Solubility of the problem (1), (2) is directly related to lower bound of small denominators $\det R(k)$ and $\det \Phi(k)$ for $k \in \mathbb{Z}^m$, where $\Phi(k)$ is Vandermonde matrix generated by numbers $\exp(\lambda_j(k) - \lambda_1(k))\Delta t$, $\lambda_j(k)$ is the root of characteristic equation

$$\det(I\lambda^n + L_1(k)\lambda^{n-1} + \ldots + L_n(k)) = 0,$$

$$R(k) = col(\alpha(k), \alpha(k)A(k), \dots, \alpha(k)A^{np-1}(k)),$$

$A(k) = (A_{ij}(k))_{i,j=\overline{1,n}}$, $A_{i,i+1}(k) = I(i = \overline{1, n-1})$, $A_{nj}(k) = L_{n-j+1}(k)$, $A_{ij}(k) = 0$ otherwise.

Estimates of small denominators are established by means of metric theory of Diophantine approximations.

On Inhomogenous Simultaneous Diophantine Approximation

Asmus L. Schmidt Jin Yuan and L. Wang

Department of mathematics, Northwest University, Xián, P. R. China phone: (0086) 29423052

Recently, R. Kannan and L. Lovász presented some results on Kronecker's theorem on diophantine approximation in the real and inhomogenous case. In 1955, É. Lutz considered the p-adic case. In the present paper we give a more general form of Kronecker's theorem and use the Basic Reduction Algorithm of Lenstra, Lenstra and Lovásh, to solve the problem of inhomogenous diophantine approximation in the real and p-adic cases.

On the number of integer polynomials with the limitation on derivative at a root

N. Kalosha

Belarus State University, Minsk

Let $F_n(N,p)$ be the set of leading polynomials P with $\deg P \leq n$, $H(P) \leq N$ and for some complex root of P α the inequality $|P'(\alpha)| < N^{1-p}$ holds.

R.Baker proved that $\#F_n(N,p) < c_1(n)N^{n+1-p}$ for $0 < p \leq 1$.

Theorem.$\#F_3(N,p) < c_2(n)N^{4-2p}$ *for* $0 < p \leq \frac{1}{2}$

Sums of fractional parts for the functions of special kind

A.A. Karatsuba

Russia, 117234, Moscow, Lenin Hills, MGU, korp. I, 50, phone: 9390519

We proved the theorems on the distribution of fractional parts of the function $(ax^* + bx)/m$, where $m \geq m_1 > 0$, m is an integer, $(a,m) = 1$, b is an integer, $(x,m) = 1$, $x^*x \equiv 1 \pmod m$, $0 < x \leq X$. Here X can be very small as compared to m, e.g. $X \leq m^\varepsilon$, $\varepsilon > 0$ is any fixed number, $m \geq m_2(\varepsilon) > 0$.

Representation of numbers by quadratic forms in connection with the theory of modular forms.

L.A. Kogan

Uzbekistan, 700123, Tashkent, Chilanzar, kvartal 12, 5, 24

We show some applications of the results obtained by author in connection with the conjecture of A. Weil to the problems of representation of numbers by quadratic forms.

We also present some results of the author connected to the generalization of his methods in the theory of quadratic forms. Here we take advantage of the generalized theta-functions with characteristics which were used earlier by T.V. Vephvadze to generalize the methods of G.L. Lomadze.

To the proof of great Fermat's theorem by the method of analytical isometries in locally compact nonarchimedean fields

G.I. Gusev, O.A. Koroleva

Russia, 410076, Saratov, Chernyshevskogo 19, 48

It is well known that there is a bijective correspondence between all non-zero and non-unit prime divisors of ring of integers for any finite extension of $\mathbb{Q}$ and the classes of equivalent non-archimedean valuation.

Let l be an odd prime and p be a prime such that $p \equiv 1 \pmod l$. Then the polynomial $\Phi_l(x) = \frac{x^l - 1}{x-1}$ is a product of pairwise distinct linear multipliers in the field of $p-$adic numbers $\mathbb{Q}_p$. Therefore the principal divisor (p) of the ring of integers for the field $\mathcal{Q}(\zeta)$ is non-ramified and has the decompozition $(p) = \mathcal{P}_1 \dots \mathcal{P}_{l-1}$, here ζ is a primitive root of degree l of 1, $\mathcal{P}_j$ are pairwise distinct prime divisors, $j = 1, \dots, l-1$ ([1],p. 255-259).

As a corollary we have the canonic decomposition of Kummer for the divisors of cyclotomic field of the type $(a - b\zeta^s)$. This decomposition is connected to Fermat's equation $A^l - B^l = C^l$, where a, b are any coprime integers and $s = 1, \ldots, l-1$. The metrical properties of the coefficients of l–adic analog for Newton series allow to obtain this result. These decompositions are used in the proof of the first case of Fermat's theorem ([2], p. 184-186).

Using the modification of Eisenstein irreducibility criterion one shows that there is a unique extention of p–adic norm over cyclotomic field $Q_l(\zeta)$ whence it follows that $(l) = \mathcal{L}^{l-1}$, where $\mathcal{L}$ is a prime divisor of the type $\mathcal{L} = (1-\zeta)$

Morover with the properly chosen analitical isometry on some l–adic sphere of the field $Q_l(\zeta)$ the polynomial $y^{l-1} + ly^{l-2} + \ldots + l$ is mapped into the polynomial $z^{l-1} + l$. The latter polynomial is used for the proof of the second case of Fermat's theorem for regular prime exponents ([2], p 431-435).

The technique of isometrical transformations allows to choose the proper basis of "real" Kummer field , which facilitate the proof of some classical results.

The new estimate for the linear forms of logarithms of algebraic numbers

S. Kotov

Belarus, Minsk, ul. Kupaly 23-51

We prove the following result

Theorem: *Suppose $\alpha_1, \ldots, \alpha_n, \beta$ are algebraic numbers of the degree no greater then g all distinct from 0 and 1; $h_1, \ldots, h_n$ are rational integers, $H = \max_{1\le i\le n}(|h_i|)$, $\sigma > l$ is a value effectively defined in terms of n, g and the height of numbers α_i, $1 \le i \le n$. If*

$$\exp\{-\sigma^{-1}H\} \le |\beta| \le \exp\{\sigma^{-1}H\}$$

and

$$h(\alpha_1^{h_1}, \ldots, \alpha_n^{h_n}) \le \exp\{\sigma^{-1/2}\ln\sigma H\} \quad (1)$$

then

$$|\beta - \alpha_1^{h_1}, \ldots, \alpha_n^{h_n}| > \exp\{-\varepsilon H\} \quad (2)$$

for $4\sigma^{-1/2}(\ln\sigma)^{3/2} < \varepsilon < 1$ with the condition that the right part of (2) is nonzero.

Clearly, instead of inequalities (2) we may consider the analogous inequalities for linear forms

$$\ln\beta - h_1\ln\alpha_1 - \ldots h_n\ln\alpha_n$$

of logarithms of algebraic numbers $\alpha_1, \ldots, \alpha_n, \beta$ (we take arbitrary but fixed branch of the logarithm). It is well known that the estimates (2) are widely used for the effective analysis of diophantine equations.

The estimates of the type (2) obtained before essentially used the height of β. In our case we need only lower and upper bounds for $|\beta|$ under the condition (1). However for the effective analysis of diophantine equations this condition is not essential.

The method of trigonometric sums in the theory of Diophantine approximations of dependent quantities

E. Kovalevskaya

Institute of Mathematics, Academy of Sciences of Belarus, 220072, Surganova 11, Minsk, Belarus

e-mail: *imanb%imanb.belpak.minsk.by@demos.su*

This method is one of the most powerful tools in Analytic Number Theory. For the first time in 1949 J.P. Kubilius applied it to the solution of a problem in the metric theory of numbers. Later, in 1962-1977, V.G. Sprindžuk used this to obtain many results related to the extremal manyfolds of large dimension.

Let Γ be a manifold in $\mathbb{R}^n$, $\Gamma = (f_1(\bar{x}), \ldots, f_n(\bar{x}))$, where $f_i(\bar{x}) \in C(X)$, $(1 \le i \le n)$, $\bar{x} = (x_1, \ldots, x_m) \in X \subset \mathbb{R}^m$, $m < n$. Sprindžuk's approach counts the rational points near Γ, from which we obtain the sums of the form:

$$S = \sum_{m_1}\cdots\sum_{m_n}\sum_{q=1}^{Q} e^{2\pi i q(m_1 f_1(\bar{a}/q)+\ldots+m_n f_n(\bar{a}/q))},$$

where $m_i \in \mathbb{Z}$ $(1 \le i \le n)$, $\bar{a} \in \mathbb{Z}^m$, $0 < a_j \le q$, $(1 \le j \le m)$. It is desirable to obtain such upper estimates of $|S|$ that the extremality of Γ is provided for a broad class of smooth functions $f_1(\bar{x}), \ldots, f_n(\bar{x})$ with $m \ge n/2$. A set of other well approximable points on the extremal manifolds has zero Lebesque measure. Their detailed investigation is performed in terms of Hausdorf dimension, for which a lower estimate of $|S|$ is required.

We prove a Lemma for getting the asymptotic estimate of $|S|$ by the method of trigonometric sums.

Lemma on p-adic linearisation and hua's lemma on partition

M.V.Kudrjavtsev

Russia, Saratov, 410038, Sokolovogorskaua, 15, 1

Let $p > 2$ be a prime, $\alpha \geq 2$ be an integer, $\mathbf{O}_p$ be a ring of p-adic integer numbers,

$$F = F(X_1, \ldots, X_n) = \sum_{i_1 \geq 0} \cdots \sum_{i_n \geq 0} a_{i_1,\ldots,i_n} X_1^{i_1} \cdots X_n^{i_n} \in \mathbf{O}_p[[X_1, \ldots, X_n]]$$

be a power series such that $\lim\limits_{|\bar{i}| \to +\infty} | a_{i_1,\ldots,i_n} |_p = 0$, where $| \bar{i} | = i_1 + \cdots + i_n$.

The modification of Hua's lemma on partition is aplied at upper estimating the complete rational trigonometric sum

$$\mathbf{S} = \sum_{1 \leq x_1 \leq p^\alpha} \cdots \sum_{1 \leq x_n \leq p^\alpha} \exp\left(2\pi i \frac{F(x_1, \ldots, x_n)}{p^\alpha}\right).$$

One of the applications is obtained when the intervals of summating for $x_i: \; 1 \leq x_i \leq p^\alpha \quad (1 \leq i \leq n)$ are partitioned by replacement $x_i = y_i + p^{s_i} z_i \quad (1 \leq i \leq n)$.

It is found that Hua's lemma and its modifications are the result of application of the lemma on the isometric linearization [1].

In the simplest case we suppose that $s_i = s = [\alpha/2] \quad (1 \leq i \leq n)$ (for $n = 1$ Smith [2]). Let $F_i = \frac{\partial F}{\partial x_i}$ and $N(\overline{F}, s)$ be a number of solutions of the system of congruence

$$F_i(x_1, \ldots, x_n) \equiv 0 \,(\mathrm{mod} p^s), \quad 1 \leq x_i \leq p^s \quad (1 \leq i \leq n). \qquad (1)$$

For fixed $\bar{x} = (x_1, \ldots, x_n) \in \mathbf{O}_p^n$, we denote by $r = r(\bar{x})$ the number of zeros among p-adic exponents constituting the system of p-adic invariants of the matrix $\left(\frac{\partial^2 F}{\partial x_i \partial x_j}\right)_{1 \leq i,j \leq n}$ of the quadratic form (see [3], p.131) at point $\bar{x}$. The set of solutions of the system (1) is partitioned on subsets so that in k-th $(0 \leq k \leq n)$ subset there are solutions $\bar{x}$ iff that $r(\bar{x}) = k$ and let $N_k(\overline{F}, s)$ be a number of those solutions.

Theorem. The following estimation is valid

$$| \mathbf{S} | \leq \sum_{k=0}^{n} N_k(\overline{F}, s) p^{\frac{\alpha k}{2} + (\alpha - s)(n-k)}.$$

In particular if for all $\bar{x} \in \mathbf{O}_p^n$ Hessian $\det \left(\frac{\partial^2 F}{\partial x_i \partial x_j}\right)_{1 \leq i,j \leq n}$ is p-adic unit, then

$$| \mathbf{S} | \leq N(\overline{F}, s) p^{\frac{\alpha n}{2}}.$$

References:

1. Гусев Г.И., Кудрявцев М.В. Изометрическая линеаризация многочленов в p-адических полях и ее приложения //Сарат. гос. ун-т.-Саратов.-1990.-25 с. Деп. в ВИНИТИ 11.04.90, N 20923-B90.

2. Smith R.A. Estimates for exponential sums// Proceed Amer. Math. Soc.- 1980.- V. 79, N. 3.-P. 365-368.

3. Cassels J.W.S. Rational quadratic forms. London, Academic Press, 1978.

Value-distribution of the Lerch zeta-function

A. Laurinčikas

Lithuania, Vilnius, Naugarduko, 24, Vilnius Univ., Facult. Math.

Let $s = \sigma + it$ be a complex variable, and let $\mathbb{Z}$ denote the set of integer numbers. We consider the function

$$L(\lambda, \alpha, s) = \sum_{m=0}^{\infty} \frac{e^{2\pi i \lambda m}}{(m + \alpha)^s}, \quad \sigma > 1,$$

introduced by M. Lerch in 1887. Here λ and α are real numbers, $0 < \alpha \leq 1$. When $\lambda \notin \mathbb{Z}$ the function $L(\lambda, \alpha, s)$ is analytically continuable over the s-plane to an entire function. In this case we prove a limit theorem in the sense of weak convergence of probability measures in the complex plane. When α is a transcendental number such a theorem for $L(\lambda, \alpha, s)$ is proved in the space of analytic functions equipped with the topology of uniform convergence on compacts. Finally, the latter theorem is used to derive the universality property of the Lerch zeta-function.

Explicit lower estimate for rational homogeneous linear forms in logarithms of algebraic numbers

E. Matveev

127018, ul. Oktjabrskaya 38–34, Moscow, Russia

Let $\mathbb{K}$ be a number field of degree $D_{\mathbb{K}}$ over $\mathbb{Q}$ embedded into $\mathbb{C}$. If $\mathbb{K} \subseteq \mathbb{R}$ then put $\kappa = 1$ othewise put $\kappa = 2$, $D = D_{\mathbb{K}}/\kappa$. We are given $\alpha_1, \ldots, \alpha_n \in \mathbb{K}^*$, $n \geq 2$, satisfying the condition

$$[\mathbb{K}(\sqrt{\alpha_1}, \ldots, \sqrt{\alpha_n}) : \mathbb{K}] = 2^n$$

and having the absolute logarithmic heights $h(\alpha_j)$, $(1 \leq j \leq n)$. Let $\log \alpha_1, \ldots, \log \alpha_n$ be any fixed determinations of logarithms. Put

$$\rho = \mathrm{rank}_{\mathbb{Z}}\{\log \alpha_1, \ldots, \log \alpha_n\}.$$

We consider the linear form $\Lambda = b_1 \log \alpha_1 + \ldots + b_n \log \alpha_n$ with $b_1, \ldots, b_n \in \mathbb{Z}$, $b_n \neq 0$.

Theorem. *If we put*
$C_1 = (2 + n \log 2)(n+1)\rho/n^2$,
$C_2 = 4(n+1)(6 + 5/(n \log 2 + 2))e^{2n}\rho/n^{3/2}$,
$A_j = \max\{h(\alpha_j),\ |\log \alpha_j|/D, 1/DC_1\}$, $(1 \leq j \leq n)$,
$\Omega = A_1 \cdots A_n$, $\omega = \Omega(C_1 D)^n (ne/2\rho)^\rho$,
$B = \max\{|b_j| A_j/A_n : 1 \leq j \leq n\}$,
$C_0 = \log(C_2 D\omega/C_1 A_n)$, *then*

$$\log|\Lambda| > -112 \cdot 2^n C_2 C_0 D^2 \omega \log(2eB).$$

R e m a r k. The main improvement of this theorem is the absence of the multiple n^n in the estimate.

On some Diophantine equations connected with Pellian equation

D. A. Mitkin

Russia, Moscow, Moscow Pedagogical state Univ.
e-mail: mitkin.nt@mpgu.msk.su

Let D be a positive integer which is not a perfect square and $4 \nmid n$. Then there are exactly two positive integerly solvable equations in the set of Diophantine equations

$ax^2 - by^2 = c$, $a, b \in \mathbb{N}$, $ab = D$, $(a,b) = 1$, $1 \leq a < b$, where $c = \pm 1$ if D is even and $c = \pm 1, \pm 2$ if D is odd. One of them is the Pellian equation $x^2 - Dy^2 = 1$ and second one has $c = \pm 1$ in the cases $D \equiv 1(mod4)$ or $D \equiv 2(mod4)$ and $c = \pm 1$ or $c = \pm 2$ in the case $D \equiv 3(mod4)$. The integral quadratic forms

$\pm(ax^2 - by^2)$, $a, b, \in \mathbb{N}$, $ab = D$, $(a,b) = 1$, $1 \leq a < b$,

can be partitioned into the pairs of integerly equivalent forms if the right part c of the second solvable equation has the modulus 1 or they are integerly equivalent if the right part c of the second solvable equation has the modulus 2.

Draxl's zeta-functions and integer points of affine toric varieties

B.Z. Moroz

Mathematical Institute, 24-29 st Giles, Oxford Ox1 3Lb, England (go 31/I/97)
Max-Plank -Institut für Math., Gottfried-Claren-Str. 26, D-5325 Bonn, Germany
moroz@mpim-bonn.mpg.de

Given an affine toric variety X over a number field k, one defines a class of toric varietes $X_{\mathfrak{B}}$ over the ring of integers $\mathfrak{o}$ of k indexed by certain sequences $\mathfrak{B}$ of fractional ideals of k. We study the distribution of integer points $X_{\mathfrak{B}}(\mathfrak{o})$ in the real locus $X(\mathbb{R})$ (by construction, $X_{\mathfrak{B}}(\mathbb{R})$ may be identified with $X(\mathbb{R})$). I prove an asymptotic formula for the number of integer points in a large "cube" in $X(\mathbb{R})$. Morover, under certain restrictions on X, one can prove an equidistribution theorem, an asymptotic formula for the number of integer points in an arbitrary "smooth subset" of $X(\mathbb{R})$. These results generalize and strengthen my results on the number of integer points on norm-form varieties described elsewhere (cf. Quarterly J. of Math. (Oxford), 45(1994), 243-253). The L-functions defined in the late sixties by R.K.J. Draxl provide an essential analytic tool used for the solution of this counting problem. I refer to the following series of my recent papers for details:

1. Exercises in analytic arithmetic on an algebraic torus, The F.Hirzebruch Festband, Israel Math.Conference Proceedings, 9 (1996), 347-359.

2. On the integer points of some toric varieties, Quarterly J.of Math. (Oxford), 48 (1997), to appear.

3. On the distribution of integer points in the real locus of an affine toric variety, Proc.of the Symposium on Sieve methods,

Exponential Sums, and their applications in Number Theory, to appear.

4. On the integer points of an affine toric variety (general case), submitted for publication.

On the structure of the set of small values for the polynomials with increasing height

I.M. Morozova *prosp. Skoriny, 99, BATU*

Let $P(z) \in \mathbb{Z}$ be a polynomial of the degree n, with the height $H = H(P)$. Let $\varepsilon(H)$ be a positive monotonously decreasing function and $\sigma_\varepsilon(P)$ be a linear domain in $\mathbb{C}$ such that for any $z \in \sigma_\varepsilon(P)$ the inequality $|P(z)| < \varepsilon(H)$ holds true. For primitive polinomials and for $\varepsilon(H) = H^{-w}$, $w > n-1$ it was proven by V.G. Sprindžuk that

$$\text{mess } \sigma_\varepsilon(P) > c(n)(\text{diam } \sigma_\varepsilon)^2 \tag{3}$$

Theorem: *The inequality (1) is true for any polynomial $P(z)$ and any function $\varepsilon(H)$.*

On the best simultaneous approximations.

N.G. Moshchevitin

117234, Russia, Moscow, Lenin Hills, central building of MSU, Dept. Number Theory, e-mail: mosh@nw.math.msu.su

Let $\alpha_1, \ldots, \alpha_s \in \mathbb{R}$ be linearly independent over $\mathbb{Z}$ togather with 1.

Best simultaneous approximations $(p_\nu, a_{1\nu}, \ldots, a_{s\nu})$ are defined as relative minimums of the form $\max\limits_j ||p\alpha_j|| = \max\limits_j |p\alpha_j - a_j|$.

Theorem 1: *There exists a continuum set of α for which the best simultaneous approximations satisfy the following equality:*

$$\text{rk} \begin{pmatrix} p_\nu & a_{1\nu} & \ldots & a_{s\nu} \\ & \cdots & \cdots & \\ p_{\nu+s} & a_{1\nu+s} & \ldots & a_{s\nu+s} \end{pmatrix} = 2 \text{ or } 3$$

Theorem 1 gives a counterexample to J. Lagarias conjecture.

The best approximations $m^\nu \in \mathbb{Z}^{s+1}$ in the sense of linear form are defined as the relative minimums of the form $||m_1\alpha_1 + \ldots + m_s\alpha_s|| = |m_1\alpha_1 + \ldots + m_s\alpha_s + m_0|$.

Theorem 2: *There exists a continuum set of α and a linear subspace $\mathcal{L}_\alpha$, $\dim \mathcal{L}_\alpha = 3$ such that for all sufficiently large ν we have $m^\nu \in \mathcal{L}_\alpha$*

Theorem 2 is proved in terms of singular Khinchine systems. We obtain some generalizations of singular Khinchine systems in terms of best approximations.

Sets of integers of positive density and recurrence

R.Nair

Dept. Math. Sciences, University of Liverpool,
Liverpool, England
e-mail: nair@liv.ac.uk

Suppose $(k_n) \subseteq \mathbb{N}$ is "strongly Hartman uniformly distributed", for $S \subseteq \mathbb{N}$. We set $d(S) = \lim\limits_{N\to\infty} |\frac{S\cap[1,N]}{N}|$ (when defined) and we set $b(S) = \sup\limits_I \lim\limits_{N\to\infty} |\frac{S\cap I_n}{I_n}|$, where the supremum is taken over all $I = (I_n)$ (interval) with $|I_n|$ tending to infinity. Then given $E \subseteq \mathbb{N}$ with $b(E) > 0$ there exist $R \subseteq \mathbb{N}$ with $d(R) \geq b(E)$ such that for every finite set $\{n_1, \ldots, n_r\} \subseteq R$ we have

$$b(E \cap (E + k_{n_1}) \cap \ldots \cap (E + k_{n_r})) > 0$$

Sequences (h_n) covered by the hypothesis of being "strongly Hartman uniformly distributed" include $k_n = [n^\alpha]$ ($\alpha \notin \mathbb{N}$, $\alpha \geq 1$) and $k_n = [e^{(\log n)^\gamma}]$, $\gamma \in (1, 3/2)$.

Modular functions and algebraic independence

Yu. V. Nesterenko

Moscow State University
E-mail: nest@nw.math.msu.su

Let $P(z)$, $Q(z)$, and $R(z)$ be the functions introduced by S. Ramanujan in 1916, let $J(z) = 1728Q(z)^3/(Q(z)^3 - R(z)^2)$, and let $j(\tau) = J(e^{2\pi i\tau})$ be the modular invariant.

Theorem 1. *For each $q \in \mathbb{C}$, $0 < |q| < 1$,* there exist at least three numbers algebraically independent over $\mathbb{Q}$ among $q, P(q), Q(q)$, and $R(q)$.

Corollary 1. *Let q, $0 < |q| < 1$,* be an algebraic number. Then the numbers in each set $\{P(q), Q(q), R(q)\}$ and $\{J(q), \theta J(q), \theta^2 J(q)\}$

are algebraically independent. (Here $\theta = z\frac{d}{dz}$.) In particular, all these numbers are transcendental.

The assertion about the algebraic independence of the numbers in the second set was conjectired by D. Bertrand in 1977.

Corollary 2. *Let $\wp(z)$ be the Weierstrass elliptic function with algebraic invariants g_2 and g_3 and with complex multiplication over some field k*; let ω be a period of $\wp(z)$. Then the numbers π, ω, and $e^{2\pi i \tau}$ are algebraically independent for each $\tau \in k$, $\operatorname{Im}\tau \neq 0$. In particular, the numbers in each of the following sets: 1) $\{\pi, e^{\pi}, \Gamma(1/4)\}$; 2) $\{\pi, e^{\pi\sqrt{3}}, \Gamma(1/3)\}$; 3) $\{\pi, e^{\pi\sqrt{D}}\}$, $D \in \mathbb{Z}$, $D > 0$, are algebraically independent.

The assertion about the algebraic independence of π and e^{π} is one of the oldest conjectures in the theory of transcendental numbers.

Theorem 2. *Let q, $0 < |q| < 1$, and θ_1, θ_2, and θ_3 be* complex numbers such that $q, P(q), Q(q)$, and $R(q)$ are algebraic over $\mathbb{Q}(\theta_1, \theta_2, \theta_3)$. Then there exists a constant $\gamma > 0$, which depends only on q and θ_i, such that for each non-zero polynomial $A \in \mathbb{Z}[x_1, x_2, x_3]$,

$$|A(\theta_1, \theta_2, \theta_3)| > \exp\left(-\gamma t(A)^4 \log^{24} t(A)\right), \qquad (*)$$

where $t(A) = \log H(A) + \deg A$. In particular, such an estimate holds for the sets 1) and 2) of Corollary 2.

The value 4 of the exponent on the right-hand side of $(*)$ is best possible.

Normal numbers and Riesz products

A. Pollington

Brigham Young University, Provo, Utah, USA
e-mail: andy@math.byu.edu

We give a caracterization of when normality in one base is equivalent to normality in another. By normality, we mean that, for x $x\theta^n$ is uniformly distributed modulo one, $\theta > 1$ is a real number, not necessarily an integer, then x is normal in base θ. This is a joint work with Gavin Brown and William Horan and answers the questions posed by Michel Mendes (France).

Divisibility sequences and other Hadamard Problems

A.J. van der Poorten

Center for Number Theory Research, Macquarie University, Sidney 2109 Australia

It is well known that the sequence (f_h) of Fibonacci numbers, defined by $f_{h+2} = f_{h+1} + f_h$ and the initial values $f_0 = 1$, $f_1 = 1$ is a ***divisibility sequence***: whenever $h|k$ also $f_h|f_k$. That this is so is easy to see. If α and β are the positive and negative zeros respectively of $X^2 - X - 1$ then $f_h = (\alpha^h - \beta^h)/(\alpha - \beta)$. Morgan Ward had asked whether a third order (ternary) recurrence sequence of integers (a_h), thus one given by a power sum $a_h = A_1\alpha_1^h + A_2\alpha_2^h + A_3\alpha_3^h$, with distinct roots (α_i) and nonzero constant coefficients A_i, can be a divisibility sequence if it is essentially different from the square of a binary divisibility sequence.

I settled that seemingly intractable diophantine problem ('A full characterisation of divisibility sequences') (with J.-P. Bézivin and A.Pethö), *Amer. J. Math.* **112** (1990), 985-1001) by noticing that it (and a quite considerable generalization of it) unexpectedly followed from a factorisation theory for exponential polynomials, and the following theorem of mine ('Solution de la conjecture de Pisot sur le quotient de Hadamard de deux fraction rationelles', *C.R. Acad. Sc. Paris (Série 1)* **306** (1988), 97-102):

If both $\sum_{h\geq 0} a_h b_h X^h$ and $\sum_{h\geq 0} b_h X^h$ represent rational functions, and the a_h all are integers (or, even, all belong to a ring of finite type), then $\sum_{h\geq 0} a_h X^h$ represents a rational function.

This 'Hadamard quotient' theorem allows a lifting of the arithmetic conditions to the function case.

Ther are related problems that in effect call for a generalisation of the Hilbert Irreducibility Theorem. An example is Pisot's Conjecture whereby *if $\sum_{h\geq 0} a_h^3 X^h$ represents a rational function and the a_h all are integers then also $\sum_{h\geq 0} a_h X^h$ represents a rational function.*

alf@mpce.mq.edu.au

Small denominators in the problems of mathematical physics

B. Ptashnyk

Ukraine, L'viv
e-mail: Ptashnik% ims.ipam.Lviv.ua@litech.Lviv.ua

The investigation of the problems with two- and multi-point time conditions for the equations

$$\sum_{|s|\le n} A_s \frac{\partial^{|s|}u(t,x)}{\partial t^{s_0}\partial x_1^{s_1}\ldots\partial x_p^{s_p}} = f(t,x),\ t\in(0,T),\ x\in\Omega_p \quad (4)$$

where $A_s \in \mathbb{C}$, Ω_p is a p-dimensional torus, is connected with the problems of small denominators. In particular, in the problem with conditions

$$\sum_{r=0}^{n-1} a_r \frac{\partial^r u(t_j,x)}{\partial t^r} = \phi_j(x),\ j=\overline{1,n};\ 0\le t_1<\ldots<t_n\le T;\ a_r\in\mathbb{C} \quad (5)$$

the small denominators appear in such form:

$$\Delta(k) \equiv \det\|\exp(\lambda_q(k)t_j)\|_{j,q=1}^n$$

$$\alpha_q(k) \equiv \sum_{r=0}^{n-1} a_r(\lambda_q(k))^r,\ \beta_q(k) \equiv \prod_{l=1,\,l\ne q}^{n}(\lambda_q(k)-\lambda_l(k)),$$

where $\lambda_q(k), q=\overline{1,n}$ are the roots of the equation

$$\sum_{|s|\le n} A_s\lambda^{s_0}(ik_1)^{s_1}\cdots(ik_p)^{s_p} = 0,\ k=(k_1,\ldots,k_p)\in\mathbb{Z}^p\backslash\{(0)\}.$$

It is proved that for all $k\in\mathbb{Z}^p$, nearly all (relative to Lebesque measure) vectors composed of real and imaginary parts of the coefficients A_s, a_r and for nearly all vectors $(t_1,\ldots,t_n)\in[0,T]^n$ the following estimates are valid: $|\Delta(k)| \ge C_1\|k\|^{-\gamma_1}\exp(-\gamma_0\|k\|)$, $|\alpha_q(k)| \ge C_2\|k\|^{-\gamma_2}$, $|\beta_q(k)| \ge C_3\|k\|^{-\gamma_3}$, where c_j, γ_j are positive numbers independent of k and $q=\overline{1,n}$.

In case of hyperbolic equation in (1) and equidistant points t_j, $j=1,\ldots,n$ in (2) similar results were obtained by the author jointly with Prof. Bernik.

Monoidal rings of focus quantities

V.G. Romanovsky

Belarusian State University of Informatics and Radioelectronics
P.Brovka,6 Minsk, 220027, Belarus, e-mail:math@micro.rei.minsk.by

Two problems, closely related to the second part of the 16-th Hilbert problem, namely, the centre-focus problem and the problem of bifurcation of small-amplitude limit cycles from a singular point $w=0$ of the polynomial vector field

$$i\frac{dw}{dt} = w - i\lambda w - \sum_{\substack{j+l=2\\ l,j\ge 0}}^{n} a_{l-1,j}w^l\overline{w}^j. \quad (6)$$

where $\lambda\in\mathbf{R}, a_{l-1,j}\in\mathbf{C}$ are considered. We show, that these problems can be considered as purely algebraic problems: we give a recurrent formula to compute polynomials (called *focus quantity*) which generate an ideal J, and then it is necessary to find a basis of the ideal. Thus investigating the cyclicity or centre-focus problem it is useful to be able to solve the ideal membership problem and it is also necessary to know how to obtain the simplest description of the variety (the set of common zeros of all polynomials of J) $V(J)$ of the ideal J.

We consider focus quantities as elements of special monoidal rings, associated with the monoid, formed by solutions of the diophantine equation

$$L(\nu) = \begin{bmatrix}1\\0\end{bmatrix}\nu_1 + \begin{bmatrix}0\\1\end{bmatrix}\nu_2 + \cdots + \begin{bmatrix}-1\\n\end{bmatrix}\nu_l + \begin{bmatrix}n\\-1\end{bmatrix}\nu_{l+1} + \cdots$$

$$+\begin{bmatrix}0\\1\end{bmatrix}\nu_{2l} = \begin{bmatrix}m\\m\end{bmatrix},$$

$\nu=(\nu_1,\nu_2,\ldots,\nu_{2l})$, $\nu_k\ge 0$, $m=0,1,2,\ldots$ and the vector $\begin{bmatrix}i\\j\end{bmatrix}$ corresponds to the coefficient a_{ij} of (6). We investigate properties of these rings, which are useful for solving of considered problems.

On some complex sets with good approximation of zero by the integer polynomials.

N.V Sakovich

Math. Dept., Mogilev State Pedagogical Institute, Mogilev, Belarus

Let $L_n(w)$ be the set of real numbers for which the inequality

$$|P(x)| < H(P)^{-w} \quad (7)$$

has an infinite number of solutions in polynomials $P(x) \in \mathbb{Z}[x]$, $w > n$, $n = \deg P$. V.I. Bernik has proven that $\dim L_n(w) = (n+1)/(w+1)$

Theorem: Let $P \in \mathbb{Z}[z]$, $z \in \mathbb{C}$. Then for any real $w \in ((n-1)/2, 4n+3) \cup (30n+29, \infty)$ we have $\dim L_n(w) = (n+1)/(w+1)$.

A new criteria of the algebraic independence of the values of hypergeometric E-functions

V.Kh. Salikhov

Russia, Bryansk, Komsomolskaya 18-185
fax: (0832)56-05-33

We consider the general hypergeometric E-function

$$\phi(z) = \sum_{n=0}^{\infty} \frac{(\nu_1)_n \cdots (\nu_l)_n}{(\lambda_1)_n \cdots (\lambda_{t+l})_n} \left(\frac{z}{t}\right)^{tn},$$

where $\nu_i, \lambda_j \in \mathbb{Q} \backslash \{0; -1; -2; \ldots\}$, $(\lambda)_n = \lambda(\lambda+1)\ldots(\lambda+n-1)$.

One of classical problems of Siegel-Schidlovsky's method in the theory of transcendental numbers is obtaining the algebraic independence of numbers $\phi(\alpha), \phi'(\alpha), \ldots, \phi^{(t+l-1)}(\alpha)$, when $\alpha \in \mathbb{A}$.

The aothor has obtained the corresponding criteria earlier in the case $l = 0, 6 \not| t$. In this report a new criteria in the case $l > 0, 6 \not| t$, $\lambda_{t+1} \in \mathbb{N}$, $\gamma_i \notin \mathbb{N}$ is formulated.

Ergodic theory on $SL(n)$, Diophantine approximations and anomalies in the lattice point problem

M.M.Skriganov

Steklov Mathematical Institute
St. Petersburg Department
Fontanka 27, St. Petersburging 191011, Russia
e-mail: skrig@pdmi.ras.ru

In the present paper we study the appearence of anomalously small, particularly logarithmically small, errors in the lattice point problem for polyhedra in $\mathbb{R}^n$.

In the present paper we show that the lattice point counting for a polyhedron can be reduced to simultaneous diophantine approximations for systems of linear forms associated with the set of faces of the polyhedron. Next we interpret these diophantine approximations in terms of certain flows on the homogeneous space of the n-dimensional unimodular lattices $\mathcal{L}_n = SL(n,\mathbb{R})/SL(n,\mathbb{Z})$. Thereby we can apply the methods of ergodic theory to study the behavior of the error for a polyhedron. One of our main results, given by this strategy, can be formulated as follows.

Let tP be the dilatation of a given polyhedron $P \subset \mathbb{R}^n$ by a factor $t > 0$, and let $N(tP, \Gamma)$ denote the number of points of a unimodular lattice $\Gamma \in \mathcal{L}_n$ lying inside tP. Then for almost all $\Gamma \in \mathcal{L}_n$ (with respect to the unique invariant probabilistic measure on the homogeneous space $\mathcal{L}_n$) one has the following asymptotic formula as $t \to \infty$

$$N(tP, \Gamma) = t^n \mathrm{vol} P + O((\log t)^{n-1+\varepsilon})$$

with arbitrary small $\varepsilon > 0$.

Fibre Products of Superelliptic Curves and Codes therefrom

Serguei A. Stepanov

Russia, Moskow
e-mail: stepanov@fen.bilkent.edu.tr

Let X be a smooth projective curve of genus $g = g(X)$ defined over a finite field Fq. Let $\{x_1, x_2, \ldots, x_n\}$ be a set of F_q-rational points on X, let $D_0 = x_1 + x_2 + \cdots + x_n$ and D a F_q-rational divisor on X disjoint from

D_0. If $\deg D < n$, the Goppa construction [1] provides a linear $[n, k, d]_q$-code C with relative parameters $R = k/n$ and $\delta = d/n$ satisfying

$$R \geq 1 - \delta - \frac{g-1}{n} .$$

Thus, to produce a good geometric Goppa codes one needs smooth projective curves over F_q with a lot of F_q-rational points compared to the genus. Examples of such curves are provided by fibre products of hypetelliptic curves [2].

Theorem. *Let $f_1, \dots f_s$ be pairwise coprime square-free monic polynomials in $F_q[u]$ of the same add degree $m \geq 1$ and X the fibre product given by equations*

$$z_i^2 = f_i(u), \qquad 1 \leq i \leq s .$$

(i). The genus $g = g(X)$ of the curve X is

$$g = (ms - 3)2^{s-2} + 1 .$$

(ii). If char $F_q = p > 2$, $q = p^{2\nu}$ and $s \leq q^{1/2}$, then the number N_q of F_q-rational points of X is

$$N_q = (2q^{1/2} - s)q^{1/2}2^{s-1} .$$

Corollary. *Let $p > 2$ and $q = p^{2\nu}$. There exist a linear $[n, k, d]_q$-code C of length $n = (2q^{1/2} - s)q^{1/2}2^{s-1}$ whose relative parameters $R = k/n$ and $\delta = d/n$ satisfy*

$$R \geq 1 - \delta - \frac{sq^{1/2} - 3}{2(2q^{1/2} - s)q^{1/2}} .$$

The results are extended to the case when $q = p^{2\nu+1}$ [3] and to the case of fibre products of superelliptic curves [4] given by equations

$$z_i^l = f_i(u), \qquad 1 \leq i \leq s .$$

References

[1] *Goppa V.G.*, Codes on algebraic curves, *Soviet Math. Dokl.*, 1981, **24**, 170–172.

[2] *Stepanov S.A.*, Codes on fibre products of hyperelliptic curves, *Preprint*, 1996.

[3] *Stepanov S.A.*, Ozbudak F., Fibre products of hyperelliptic curves and geometric Goppa codes, *Preprint*, 1996.

[4] *Ozbudak F.*, Codes on fibre products of superelliptic curves, *Preprint*, 1996.

Two applications of linear forms estimates.

R. Tijdeman
Leiden University, Netherlands
tijdeman@wi.leidenuniv.nl

The first application is to numbers of Mahler's type. Mahler showed that the number 0.1248163264128256... is irrational. Several authors generalized this result. In the lecture we shall show how to apply linear form estimates to obtain even more general statements.

The second application is to a problem of Gyory, Sarcozy and Stewart and concerns joint work with Stewart. We give lower bounds for the greatest prime factors of numbers of the forms $(ab + 1)(bc + 1)(ca + 1)$ and $(ab + 1)(ac + 1)(bc + 1)(cd + 1)$.

Approximation of real numbers by algebraic integers of degree ≤ 10

Kyril Tishchenko
BATU, department of mathematics,
220100, Kulman 15, 134, Minsk, Belarus

Let ξ be a real number. Let S be the set of all real algebraic numbers of degree $\leq n$, $n > 1$. Let $\lambda > 0$ be real. We consider the problem of whether there is an infinite number of $\alpha \in S$ such that $0 < |\xi - \alpha| \ll H(\alpha)^{-\lambda}$.

Assume ξ is not an algebraic number of degree $\leq n$. In 1960 E.Wirsing [1] has shown that for any n and $\lambda = (n + 3)/2$, there is an infinite number of solutions of the inequality above. We solve this problem for $n \leq 10$ with λ being a positive root of quadratic equation

$$(3n - 5)\lambda^2 + (-2n^2 - n + 9)\lambda + (-n - 3) = 0.$$

Refrences:

1. *Wirsing E.* J.Reine Angew. Math. **206** (1960), 67–77.

A sharpening of irrationality measures of certain infinite products

Keijo Väänänen

Department of Math. Sciences University of Oulu
90570 Oulu, Finland
e-mail: **kvaanane@sun3.oulu.fi**

We give sharpenings of some irrationaity measures involving the values of the q-exponential function

$$E_q(z) = \prod_{k=1}^{\infty}(1 + z/q^k), \quad |q| > 1.$$

Theorem. *Let $q \in \mathbb{Z}$ satisfy $|q| > 1$, and let α, β be non-zero rationals satisfying $\alpha, \beta \neq -q^i$, $\beta \neq \alpha q^j$, $i = 1, 2, \ldots$; $j \in \mathbb{Z}$. Then there exists a positive costant n_0 such that for any $m/n \in \mathbb{Q}$, $n > n_0$, we have*

$$|E_q(\alpha)/E_q(\beta) - m/n| > n^{-5.6463},$$

$$|E_q'(\alpha)/E_q(\alpha) - m/n| > n^{-3.9463}$$

In the case $\beta = -\alpha$ the first bound can be improved to $n^{-3.0545}$.

We can also consider p-adic valuations and more general algebraic fields $\mathbb{K}$ instead of $\mathbb{Q}$. For example, using considerations in quadratic field $\mathbb{K} = \mathbb{Q}(\sqrt{5})$, we obtain the following result concerning the Fibonacci sequence (F_k): *There exists a positive constant n_1 such that for any $m/n \in \mathbb{Q}$, $n > n_1$, we have*

$$|\sum_{k=1}^{\infty} 1/F_k - m/n| > n^{-7.8926}.$$

Extremality of curves in $\mathbb{C}^2$, $\mathbb{C}^3$ and Hausdorff dimension

D.V. Vasilyev

Institute of Mathematics, Academy of Sciences of Belarus,
220072, Surganova 11, Minsk, Belarus
e-mail: *imanb%imanb.belpak.minsk.by@demos.su*

In 1964 W. Schmidt (see [1]) proved the extremality of curves in $\mathbb{R}^2$. The recent result [2] of V. Bernik and V. Beresnevich shows extremality of curves in $\mathbb{R}^3$.

We give the direct complex analogue of these results for the curves in $\mathbb{C}^2$ and $\mathbb{C}^3$. In particular for $\mathbb{C}^3$ we prove

Theorem 1. *Suppose functions $f_1(z)$, $f_2(z)$, $f_3(z)$ are analytical in a domain $D \subseteq \mathbb{C}$, and the Wronskian $W(f_1', f_2', f_3') \not\equiv 0$ for all $z \in D$. Denote by $A(w)$ the set of those $z \in D$ for which the inequality*

$$|a_0 + a_1 f_1 + a_2 f_2 + a_3 f_3| < (\max_{0 \leq i \leq 3} |a_i|)^{-w}$$

has infinite number of solutions in integers a_0, a_1, a_2, a_3. Then for any $w > 1$ the Lebesque measure of A equals 0

Further refinements can be made in terms of Hausdorff dimension:

Theorem 2. *For the Hausdorff dimension of $A(w)$, $w > 1$ we have the estimate*

$$\frac{4}{w+1} \leq \dim A(w) \leq \min\{\frac{8}{w+1}, 2\}.$$

References.

1. Schmidt W. //Monatsch. Math., Metrische Sätze über simaltane Approximation abhänginger lyrossen. 1964, Vol 63, N2, p.p.154-166

2. V. Beresnevich, V. Bernik //Acta Arithm. On a metrical theorem of W. Schmidt, LXXV.3 (1996), pp. 219-233.

The Jarník – Besicovitch theorem and Dynamical systems.

S.L. Velani

Department of Mathematics, Imperial College,
London, SW7

For $\tau \geq 0$, let $W(\tau)$ denote the set of 'well approximable' real numbers; that is the set of numbers which lie within $q^{-\tau}$ of infinitely many rationals

p/q $(q > 0)$. The Jarník–Besicovitch theorem expresses the 'size' of $W(\tau)$ interms of its Hausdorff dimension; $\dim W(\tau) = 2/\tau(\tau \geq 2)$.

To any dynamical system $T : X \to X$ equipped with a metric, we associate a class of 'well approximable' subsets W of the phase space X. These sets consist of points x in X whose forward trajectories $T^n(x)$ land in a prescribed 'shrinking target' infinitely often. For expanding Markov maps of the interval and rational maps of the Riemann sphere, in which case X is a Julia set, we compute the Hausdorff dimension of the set W. By specialising to the Gauss map (continued fraction map), it is shown that the results for $\dim W$ yield the Jarník–Besicovitch theorem.

The study of the well approximable sets W has consequences for those exceptional sets which arise from points having ergodic averages which do not tend to the expected limit. In turn, these exceptional sets are closely related to the multifractal theory of dynamical systems. Furthermore, Khintchine type theorems for the sets W have as consequences, analogues of Sullivan's 'logarithmic law for geodesics' on hyperbolic manifolds.

On the representation of numbers by eight step quadratic forms

T. Vepkhvadze N. Tcalugelashvili

Georgia, Tbilisi, 380008, Potochnaia 11

Generalized theta-functions with characteristics are introduced. Their modular properties are used to build the bases of the special type cusp forms spaces. It gives the opportunity of obtaining formulae for the number of representations of integers by certain eight step quadratic forms.

Here is one example of this. Let $r(n; f_k)$ denote the number of representations of positive integer n by $f_k = 2\sum_{j=1}^{k} x_j^2 + \sum_{j=k+1}^{10} x_j^2$ $(k = 1,3,5,7,9)$. Then $r(n; f_k) = \rho(n; f_k) + \sum_{j=1}^{3} \alpha_j^{(k)} \nu_j(n)$, where

$$\nu_1(n) = \sum_{n=2x_1^2+\ldots+2x_5^2+x_6^2} (x_6^2 - 2x_1^2),$$

$$\nu_2(n) = \sum_{\substack{2n = x_1^2+\ldots+x_4^2+4x_5^2+2x_6^2 \\ x_1\equiv x_2\equiv x_3\equiv x_4\equiv 1 \pmod 2}} (2x_6^2 - x_1^2)$$

$$\nu_3(n) = \sum_{\substack{8n = x_1^2+\ldots+x_4^2+4x_5^2+8x_6^2 \\ x_1\equiv x_2\equiv x_3\equiv x_4\equiv 1 \pmod 2 \\ x_5\equiv 1 \pmod 2}} (8x_6^2 - x_1^2)$$

$\rho(n; f_k)$ is the "singular series", which is summed, the numbers $\alpha_1^{(k)}$, $\alpha_2^{(k)}$ or $\alpha_3^{(k)}$ are also calculated.

Diophantine Approximation in Higher Dimension

Michel Waldschmidt

Institut de Mathematiques de Jussieu Problemes Diophantiens case 247 F–75252 PARIS Cedex 05
e-mail: miw@mathp6.jussieu.fr

Denote by

$$\mathcal{L} = exp^{-1}(\overline{\mathbb{Q}}^{\times}) = \{\lambda \in \mathbb{C}; e^{\lambda} \in \overline{\mathbb{Q}}^{\times}\} \subset \mathbb{C}$$

the $\mathbb{Q}$–vector space of the logarithms of nonzero algebraic numbers. Let d, l be two positive rational integers and let $y_1, \ldots, y_l$ be elements of $\mathcal{L}^d$. For $1 \leq j \leq l$, write $y_j = (\lambda_{1j}, \ldots, \lambda_{dj})$, where, for $1 \leq i \leq d$ and $1 \leq j \leq l$, $\alpha_{ij} = e^{\lambda_{ij}}$ is a non zero algebraic number. For simplicity, assume that the following linear independence condition is satisfied:

For any nonzero $(t_1, \ldots, t_l) \in \mathbb{Z}^d$ and any nonzero $(s_1, \ldots, s_l) \in \mathbb{Z}^l$, the number

$$\sum_{i=1}^{d}\sum_{j=1}^{l} t_i s_j \lambda_{ij}$$

does not vanish.

It is known that the $\mathbb{C}$–vector space spanned by $y_1, \ldots, y_l$ in $\mathbb{C}^d$ has dimention $\geq dl/(d+l)$. We consider an explicit quantitative version of this statment. Several variations are also studied. First, one can introduce two non-negative integers d_0, d_1, with $d_0 + d_1 > 0$, in place of the positive integer d, and replace the elements $y_j \in \mathcal{L}^d$ by elements in $\overline{\mathbb{Q}}^{d_0} \times \mathcal{L}^{d_1}$. Next, one can replace the positive integer l by (l_0, l_1), and the l-tuple $(y_1, \ldots, y_l)$ by a $l_0 + l_1$-tuple $(w_1, \ldots, w_{l_0}, y_1, \ldots, y_{l_1})$, where $w_1, \ldots, w_{l_0}$ belong to $\overline{\mathbb{Q}}^{d_0+d_1}$, while $y_1, \ldots, y_{l_1}$ belong to $\overline{\mathbb{Q}}^{d_0} \times \mathcal{L}^{d_1}$. Under suitable assumptions, the dimension of the space spanned by $w_1, \ldots, w_{l_0}, y_1, \ldots, y_{l_1}$ in $\mathbb{C}^{d_0+d_1}$ is at least

$$\frac{d_1 l_1 + d_1 l_0 + d_0 l_1}{d_1 + l_1}.$$

An effective version of this result has many consequences: lower bounds for linear forms in logarithms, algebraic independence of the exponential function, quantitative density statements related to the canonical embedding of an algebraic number field,...

A generalization to commutative groups is also available.

Elimination of the multiple n^n from estimates for linear forms in p-adic logarithms and its application to the abc-conjecture

Kunrui Yu

Department of Mathematics
The Hong Kong University of Science & Tecnology
Clear Water Bay, Kowloon, Hong Kong
e-mail: makryu@usthk.ust.hk
fax: (852) 23581643

Let $\alpha_1, \ldots, \alpha_n$ $(n \geq 2)$ be non-zero algebraic numbers and K be a number field containing $\alpha_1, \ldots, \alpha_n$ with $d = [K : \mathbb{Q}]$. Denote by $\mathfrak{p}$ a prime ideal of the ring $\mathcal{O}_K$ of integers in K, lying above a prime p, by $e_\mathfrak{p}$ the ramification index of $\mathfrak{p}$, and by $f_\mathfrak{p}$ the residue class degree of $\mathfrak{p}$. For $\alpha \in K$, $\alpha \neq 0$, write $\text{ord}_\mathfrak{p}\alpha$ for the exponent to which $\mathfrak{p}$ divides the principal fractional ideal genegated by α in K. We assume that K satisfies the conditiion

$$\begin{cases} \zeta_3 \in K, \text{ when } p = 2; \\ \text{either } p^{f_\mathfrak{p}} \equiv 3 \pmod 4 \text{ or } \zeta_4 \in K, \text{ when } p > 2, \end{cases}$$

where $\zeta_m = e^{2\pi i/m}$ $(m = 1, 2, \ldots)$. Set

$$q = \begin{cases} 3, \text{ if } p = 2, \\ 2, \text{ if } p > 2. \end{cases}$$

Let $\mathbb{N}$ be the set of non-negative rational integers and set

$$u = \max\{k \in \mathbb{N} | \zeta_{q^k} \in K\}, \quad \alpha_0 = \zeta_{q^u}.$$

Define

$$h'(\alpha_j) = \max\left(h_0(\alpha_j), \frac{f_\mathfrak{p} \log p}{d}\right) \quad (1 \leq j \leq n),$$

where $h_0(\alpha)$ denotes the absolute logarithmic Weil height of an algebraic number α. Let $b_1, \ldots, b_n \in \mathbb{Z}$ and $B = \max(|b_1|, \ldots, |b_n|, 3)$. Let $\kappa \geq 0$ be a rational integer satisfying $\phi(p^\kappa) \leq 2e_\mathfrak{p} < \phi(p^{\kappa+1})$, where ϕ is the Euler's ϕ-function. Set

$$\theta = \begin{cases} (p-2)/(p-1), & \text{if } p \geq 5 \text{ and } e_\mathfrak{p} = 1, \\ p^\kappa/(2e_\mathfrak{p}), & \text{otherwise}. \end{cases}$$

Denote by w_n a sequence of positive rational numbers, the definition of which will be given below. Now we have

Theorem. *Suppose that* $\text{ord}_\mathfrak{p}\alpha_j = 0$ $(1 \leq j \leq n)$. *If* $\Xi = \alpha_1^{b_1} \cdots \alpha_n^{b_n} - 1 \neq 0$, *then*

$$\text{ord}_\mathfrak{p} \Xi < C(n, d, \mathfrak{p}) h'(\alpha_1) \cdots h'(\alpha_n) \log B,$$

where

$$C(n, d, \mathfrak{p}) = \max(q^{-n}, p^{-f_\mathfrak{p} e_\mathfrak{p} \theta}) \cdot \frac{c}{e_\mathfrak{p}\theta}(aep^\kappa)^n \cdot \frac{(n+1)^{n+2}}{(n-1)!}.$$

$$\cdot w_n \cdot \frac{p^{f_\mathfrak{p}} - 1}{q^u} \cdot \frac{d^{n+2}}{(f_\mathfrak{p} \log p)^3} \cdot \max(f_\mathfrak{p} \log p, \log(e^6(n+1)d)),$$

with

$$c = 172386, \quad a = 54, \ \textit{if } p = 2,$$
$$c = 174727, \quad a = 25, \ \textit{if } p > 2.$$

In fact we have better values for c :

	n	*2,3,4*	*5,6,7*	≥ 8
$p = 2$	c	*61194*	*76599*	*172386*
$p = 3$ *or* $p = 5$, $e_\mathfrak{p} \geq 2$	c	*26862*	*26862*	*42964*
$p \geq 7$, $e_\mathfrak{p} \geq 2$	c	*82105*	*115038*	*174727*
$p \geq 5$, $e_\mathfrak{p} = 1$	c	*44628*	*50088*	*71276*

Furthermore if $\alpha_1, \ldots, \alpha_n$ *satisfy*

$$\left[K(\alpha_0^{1/q}, \alpha_1^{1/q}, \ldots, \alpha_n^{1/q}) : K\right] = q^{n+1},$$

then $C(n,d,\mathfrak{p})$ can be replaced by $C(n,d,\mathfrak{p})/w_n$.

Definition of w_n. *Let $w_1 = w_2 = 1$. For $n > 2$, we set*

$$w_n = 4^{s-n} \cdot (2s+2)(2s+3)\cdots(s+n+1), \text{ when } p > 2,$$
$$w_n = 6^{t-n} \cdot (3t+2)(3t+3)\cdots(2t+n+1), \text{ when } p = 2,$$

where

$$s = s_n = \left[\frac{1}{4}(1+\sqrt{16n+17})\right]$$

and $t = t_n$ is the greatest rational integer such that

$$9t^3 - 8t^2 - (8n+5)t \le 2n(n+1).$$

Here is a table for the values of w_n $(3 \le n \le 10)$:

	n	3	4	5	6	7	8	9	10
$p>2$	w_n	3/2	21/8	21/4	189/16	495/16	1485/16	19305/64	135135/128
$p=2$	w_n	4/3	2	11/3	143/18	1001/54	5005/108	1190/9	11305/27

Note that $w_n/(n!/2^{n-1}) \le 2^{s-1}/s!$ when $n > 2$ for both cases $p = 2$ and $p > 2$.

Very recently C.L.Stewart and I, using the above theorem and my other estimates for linear forms in p-adic logarithms, and Baker & Wüstholz (Crelle Journal, 1993) proved the following result:
There exists an effectively computable positive constant c such that for all positive integers x, y and z with $(x,y,z) = 1$, $z > 2$ and $x + y = z$, $\log z < G^{1/3+c/\log\log G}$.

The author is supported by RGC CERG No. HKUST633/95P.

Lower estimates of polynomials of the values of E-functions

V. V. Zudilin

Moscow State University
E-mail: `wadim@ipsun.ras.ru`

Assume that the system of homogeneous linear differential equations

$$\frac{d}{dz}y_l = \sum_{j=1}^{m} Q_{lj}y_j, \quad l = 1,\ldots,m, \quad m \ge 2, \qquad Q_{lj} = Q_{lj}(z) \in \mathbb{C}(z), \tag{1}$$

has a fundamental matrix of solutions with entries homogeneously algebraically independent over $\mathbb{C}(z)$. Let $f_1(z),\ldots,f_m(z)$ be a collection

of E-functions with rational coefficients of Taylor series that satisfies the system (1). Also let $\alpha \in \mathbb{Q}\setminus\{0\}$ be an arbitrary non-singular point of this system and let $d \in \mathbb{N}$. Then there exist positive constants γ and C dependent on the E-functions in question, α, and d such that

$$\left|P\big(f_1(\alpha),\ldots,f_m(\alpha)\big)\right| > C \cdot |h_1 \cdots h_w|^{-1} H^{1-\gamma(\log\log H)^{-1/(m^2-m+2)}},$$
$$H = \max_{1\le i\le w}\{|h_i|\},$$

for each non-trivial homogeneous polynomial $P \in \mathbb{Z}[y_1,\ldots,y_m]$ of degree d. Here the $h_1,\ldots,h_w$ are the non-zero coefficients of $P(y_1,\ldots,y_m)$.

This general result can be applied to the values of generalized hypergeometric functions with rational parameters.